The Relationship of Implicit Theories to Elementary Teachers' Patterns of Engagement in a Mathematics-Focused Professional Development Setting

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As elementary teachers aim to deepen their mathematical understandings, they engage in a relearning process that involves not only revisiting but also reconstructing their knowledge. To do so, meaningful engagement in immersion and practice-based experiences is required. This exploratory case study investigated the engagement patterns of two elementary teachers with varying implicit beliefs as they participated in a professional development that focused on relearning mathematics. Data were collected on the two participants in the form of video narratives, observation protocols, and interviews. Attention was given to their patterns of engagement in collaborative group settings as the participants moved through different phases of the professional development lesson. Results indicated that the engagement patterns of the two participants closely aligned with learning behaviors described in the implicit beliefs theory. In this way, the results suggested an extension of the implicit theories model to the relearning context. Additional implications and future questions are provided.

Introduction

Professional development is the key mechanism for supporting teachers' growth of mathematical knowledge needed to support effective teaching practices (Loucks-Horsley, Stiles, Mundry, Love, & Hewson, 2010; Sowder, 2007). Relatedly, Zazkis (2011) noted that mathematics teachers are not necessarily learning content for the first time. Rather, teachers are "*relearning*, that is, learning-again what was already learned" (p. viii), when they enter professional development settings with prior knowledge of and experience with the mathematical concepts they are studying. Unfortunately, this prior knowledge can often serve as an obstacle, representing incomplete or incorrect mathematical understandings. Therefore, it is necessary to recognize that "[relearning] is not simply adding knowledge to what was previously incorporated into a learner's repertoire. It is revisiting and reconstructing this repertoire" (p. 12).

This reconstruction of knowledge is akin to Smith's (2001) notion of transformative learning. According to Smith, teachers must engage in professional development experiences that cause them to rethink their views of mathematics. To this end, teachers must collaboratively engage in tasks that provide them with the "opportunity to experience, firsthand, . . . mathematics content and processes" (Loucks-Horsley et al., 2010, p. 179), and that include opportunities to experience "productive struggle" (National Council of Teachers of Mathematics [NCTM], 2014, p. 10). This struggle involves grappling with and making sense of mathematics. Further, these professional development experiences must address the implicit theories held by teachers (NCTM, 2014).

According to the implicit beliefs theory (Dweck, Chiu, & Hong, 1995; Dweck & Leggett, 1988), personal attributes may be viewed by an individual as either malleable, indicating that the individual ascribes to an incremental theory (i.e., growth mindset), or as nonmalleable, indicating that the individual ascribes to an entity theory (i.e., fixed mindset). Individuals holding an incremental theory are willing to take risks and put forth effort as a means for growing or developing that attribute. This effort exemplifies key behaviors necessary for engaging in productive struggle. In contrast, individuals holding an entity theory will avoid performance for fear of revealing an inadequacy or disengage due to a perceived lack of ability (Boaler, 2016; Dweck, 2006; Ricci, 2013). Such behaviors can serve as barriers to engaging in productive struggle.

Recognizing the role of productive struggle, it seemed appropriate to assume that differing implicit theories are at play during the relearning process; however, our query focused on how these implicit theories might relate to engagement patterns that could be considered predictable based on the implicit beliefs theory (Dweck et al., 1995; Dweck & Leggett, 1988). Alternatively, we thought it might be the case that different engagement patterns emerge given the relearning context. Therefore, the purpose of this study was to examine how teachers with differing implicit theories engaged in professional development activities that involved wrestling with fundamental mathematical ideas. Specifically, the research question that guided our work was: How are the implicit theories held by elementary mathematics teachers related to patterns of engagement in a professional development setting designed to invoke productive struggle during the relearning process, if at all? This examination of patterns of engagement served as an existence proof, of sorts, regarding the potential role of implicit theories in professional development, thus leading to the significance of the study.

Theoretical Framework

Dweck and Leggett (1988) described a social-cognitive model of motivation and personality (i.e. the implicit theories model), that framed an emerging theory of implicit conceptions of the nature of ability within their previous work in goal orientation and behavioral patterns. This implicit theories model proved resilient, and its extensive research base continues to support work in a variety of fields including educational psychology and mathematics education (e.g., Aronson, Fried, & Good, 2002; Blackwell, Trzesniewski, & Dweck, 2007; Dupeyrat & Mariné, 2005; Good, Rattan, & Dweck, 2012; Romero, 2013). This model, the evidence supporting its generalization to other domains (Dweck et al., 1995; Dweck & Leggett, 1988), and the instrument used to measure its constructs (Dweck et al., 1995) provided the theoretical and operational foundation for this study.

Dweck and Leggett (1988) posited that an individual's implicit assumptions about the nature of an ability lead directly to the type of goals he/she pursues regarding that ability and the behaviors he/she exhibits when faced with challenges to that ability (Dweck et al., 1995; Dweck & Leggett, 1988). Two implicit theories, the entity and incremental theories, were described. Individuals assuming an entity theory tended to view attributes as fixed, uncontrollable entities and adopted performance-oriented goals to gain or avoid judgment regarding the ability. Individuals espousing an incremental theory tended to view attributes as malleable and thus subscribed to learning goals focused on improvement of the ability (Dweck, 1986; Dweck & Leggett, 1988; Elliott & Dweck, 1988). These implicit theories and their associated goal pursuits created "a framework for interpreting and responding to events" (Dweck & Leggett, 1988, p. 260) that promoted observable behavioral patterns when an ability under consideration is challenged. Maladaptive, helpless responses, characterized by lowered performance and the avoidance of challenges, were associated with entity theories and performance goals. On the other hand, incremental theories created mastery-oriented responses and learning goals that were characterized by the pursuit of challenges and persistence when faced with failure (Diener & Dweck, 1978, 1980; Dweck, 1975; Dweck & Leggett, 1988; Dweck & Reppucci, 1973).

Although the tenets of implicit theories were initially established through the characterization of an individual's own intelligence, Dweck and Leggett (1988) proposed a framework through which its generalization to other attributes and domains occurred, culminating in the validation of a simple instrument used to assess individuals' implicit theories for multiple attributes (Dweck et al., 1995). Beginning with behavioral responses to social rejection (Goetz & Dweck, 1980) and other personal attributes (Bempechat, London, & Dweck, 1991), they predicted that for any attribute of personal significance, "viewing it as a fixed trait will lead to a desire to document the adequacy of that trait, whereas viewing it as a malleable quality will foster a desire to develop that quality" (Dweck & Leggett, 1988, p. 266). Additional evidence supported the notion that the model is generalizable to traits beyond the self (Dweck & Leggett, 1988; Erdley & Dweck, 1993; Medin, 1989). This external application of the model suggested there were further observable characteristics of an individual's interactions based on their implicit theories. Entity theorists should be seen to reject change in themselves and others and draw simplified conclusions from brief experiences. In contrast, incremental theorists should be seen to encourage growth in other individuals/organizations and experience a sense of control relative to their environment (Dweck & Leggett, 1988).

Following the introduction of the implicit theories model (Dweck & Leggett, 1988), initial research related to the model highlighted the significance of considering incremental theories in students. These studies demonstrated, for example, the influence of implicit theories on the success of seventh grade students (Blackwell et al., 2007), college students (Grant & Dweck, 2003), and female and minority students (Dweck, 2008). With this connection established, researchers next developed interventions for nurturing incremental views within individuals (Aronson et al., 2002; Blackwell et al., 2007; Good, Aronson, & Inzlicht, 2003). More recently, continuing research on the brain with connections to implicit theories indicates that encountering mistakes is closely related to the formulation of new learning and indicates variation in how those of different implicit theories process mistakes (Boaler, 2016; Mangels, Butterfield, Lamb, Good, & Dweck, 2006). We recognized the likelihood of mistakes occurring during the relearning process and determined an examination of how mathematics teachers who hold different implicit theories engage in this relearning process was warranted.

Methodology

Overview of Methodology

To examine the role of implicit theories in professional development, we utilized a case study methodology, specifically a holistic, multiple-case design (Yin, 2014). Our choice of methodology was suitable given Creswell's (2013) claim that the case study methodology is appropriate when exploring "a real-life, contemporary bounded system . . . through detailed, in-depth data collection" (p. 97). Further, we viewed this as an exploratory case study, given the purpose "to identify research questions or procedures to be used in a subsequent research study, which might or might not be a case study" (Yin, 2014, p. 238).

Research Context

The cases under investigation in this study were drawn from a professional development project designed for kindergarten through sixth grade mathematics teachers. The externally funded project represented a partnership between a university and four rural school districts in a southeastern state. Although the project included meetings and demonstration lessons that were held during the academic year, the project component of focus in this report was the summer institute that occurred during the second year of the project. A total of 82 teachers voluntarily attended this summer institute. With only six males and two African Americans, the group of participants consisted primarily of Caucasian females. Participants' years of teaching experience ranged from 1 to 36 years, with an average of 10.5 years. Table 1 provides the number of participants from each school district.

Table 1
Number of Participants from Each School District

School District	Number of Participants
Partner District A	24
Partner District B	7
Partner District C	30
Partner District D	9
Other Non-partner Districts	12

Over the course of 10 days, teachers met in grade-level groups (i.e., K–2, 3–4, 5–6) and engaged in activities focused primarily on fractions and their operations, as described in the Common Core State Standards for Mathematics (Common Core State Standards Initiative, 2010). Emphasis was given to using models, both pictorial and concrete. Appendix A provides an overview of the topics for each grade band.

Throughout the institute, teachers engaged in practice-based and immersion experiences, as defined by Loucks-Horsley et al. (2010). Practice-based experiences utilized artifacts (e.g., work samples, vignettes, videos) as a means for deepening teachers' content knowledge. Discussions of artifacts focused on the students' thinking. In contrast, immersion experiences engaged teachers in problem solving, with discussions focusing on the teachers' mathematical thinking. Immersion experiences involved mathematical tasks intended to provoke productive struggle (see Appendix B for sample tasks). A typical day in the summer institute included discussion of homework problems, discussion of an assigned reading, completion in collaborative groups of mathematical tasks, and exploration of student work related to the mathematical tasks (see Table 2). Teachers sat in groups of four each day. The participants in the groups varied from one day to the next.

Table 2
Timed Activities for a Typical Day

Timeframe	Activity
9:00 – 9:30	Problem-Solving Warm-up*
9:30 – 10:30	Review of Homework*
10:30 – 11:30	Mathematical Task*
11:30 – 12:00	Engagement with Related Student Artifacts**
12:00 – 12:30	Lunch
12:30 – 1:00	Mathematical Game
1:00 – 1:45	Reading Discussion
1:45 – 2:45	Mathematical Task*
2:45 – 3:15	Engagement with Related Student Artifacts**
3:15 – 3:30	Reflection and Summary

* Immersion experience that engaged participants in problem solving.
** Practice-based experience that engaged participants in considering students' understandings/misunderstandings via work samples, video cases, and/or vignettes.

Instruments and Data Sources

Implicit theories survey. The implicit theories survey included two parts. The first part consisted of the survey questions authored by Dweck et al. (1995). These nine items addressed implicit theories in relation to intelligence, morality, and world. The second part of the survey included three items addressing implicit theories in relation to a person's mathematical abilities. In a separate study, our results indicated that within the population of teachers, implicit theories regarding mathematical ability are distinguishable from implicit theories regarding intelligence (Willingham, Barlow, Stephens, Lischka, & Hartland, 2016). Therefore, we chose to utilize these three additional items. For all 12 items on the survey, teachers indicated their level of agreement (i.e., strongly agree, agree, somewhat agree, somewhat disagree, disagree, or strongly disagree).

Interview protocols. To gain insight into participants' implicit theories and their perceptions related to the professional development experience, we created four semi-structured interview protocols to be used over the course of the 10-day summer institute. Questions addressed the participants' perceptions of their strong academic students, struggling students, and their role in working with these students. Questions also addressed the participants' perspectives of their experiences in the professional development as a means for better understanding their implicit theories. Table 3 provides an overview of the purpose of each protocol and sample questions.

Table 3
Interview Protocols with Purposes and Sample Questions

Protocol	Purpose
1	To elaborate on responses to implicit theories survey *Take a moment to look back at your survey. Can you elaborate on this theme, particularly as it relates to your students? Can you describe specific students that illustrate your thoughts?*
2	To describe strong mathematics students and those who struggle as well as how the teacher works with these students *Describe some of your students who struggle the most with math. What do they have in common? As a math teacher, what's your role with these students?*
3	To reflect on the institute thus far with attention given to the elaboration of behaviors while working in collaborative groups and ideas that were taken from the institute to support working with students; to engage in a mathematical task as a means for demonstrating mindset and mathematical understanding *Last week, you mentioned _______ regarding struggling students. Have you explored new ways so far to improve your teaching for all of the students in your math classes? Please elaborate.*
4	To reflect on the institute and provide insights regarding working with struggling students and changes in practice. *Have you gained any insights as to how you can improve your teaching for all students in your math classes? Please elaborate.*

Observation protocol. Two of the authors developed an observation protocol to aid in identifying behaviors exhibited during the professional development. In creating the protocol, the two authors began with an online summary of behaviors associated with an entity theory and an incremental theory (Online Business with Jan & Alicia, 2014). In adapting the summary for our protocol, the two authors reviewed each of the behaviors to determine 1) whether the behavior was observable and 2) whether the behavior was aligned with the literature on entity and incremental theories. Behaviors identified as not observable or not aligning with the literature were removed from the protocol. The resulting protocol included six categories: evaluating situations, responding to challenges, dealing with setbacks, putting forth effort, responding to criticism, and responding to the success of others. Sample behaviors were listed for each category that represented the two implicit theories. For example, in the category c*hallenges*, sample behaviors included *gets defensive or gives up easily* (entity) and *shows signs of encouragement, towards self and others* (incremental).

The final protocol consisted of two pages. The first page included the categories with sample behaviors. The second page consisted of a blank space in which observed behaviors could be noted under the appropriate heading (i.e., incremental theory or entity theory). The rest of the author team reviewed the protocol for face validity (Patton, 2002). In completing the protocol, attention was given to recording behaviors that reflected the two opposing implicit theories without concern for which of the categories the behavior represented.

Video. We recorded video of participants working in collaborative groups during the summer institute. Observation protocols were employed concurrently with the video.

Researchers as instruments. As described by Creswell (2013), each of us served as instruments in this study as we collected and analyzed the qualitative data. Two members of the research team hold terminal degrees in mathematics education with a third member holding a terminal degree in mathematics. These individuals have extensive background in conducting qualitative research. The remaining two members of the research team were doctoral students at the time of the study, both with previous experience in collecting and analyzing qualitative data. As a unit, our collective knowledge and experiences provide evidence of necessary qualifications to serve as instruments in this study.

Participants and the Selection Process

The selection process. After receiving approval from the university's Institutional Review Board, we aimed to select two teachers for participation in this study, one ascribing to an incremental theory and the other to an entity theory. We hypothesized that the cases would yield contrasting results, thus supporting our desire for theoretical replication (Yin, 2014). An additional criterion for selection was grade level, as we wanted the two participants to be in the same grade-level group.

To identify potential participants, on the first day of the summer institute all teachers signed consent forms and completed the implicit theories survey. We then scored these surveys following Dweck and colleagues' (1995) scoring protocol. With particular attention given to the intelligence and mathematical ability subscores, we selected four teachers with low averages, indicating an entity theory, and four teachers with high averages, indicating an incremental theory. Recognizing the limitations of Likert-scale data, we interviewed these eight teachers on the second day of the summer institute to gain insights into their responses on the survey. Based on these interviews, we selected two teachers, one with an entity theory or a fixed mindset (Ms. Fitzgerald) and one with an incremental theory or growth mindset (Ms. Gorman), to participate in our study. The reader should note that these pseudonyms were selected so that the first letter of the name corresponds with their implicit beliefs: F–fixed or entity theory and G–growth or incremental theory. Both Ms. Fitzgerald and Ms. Gorman participated in the grades 5–6 professional development group, which had as its primary focus the modeling of multiplication and division with fractions.

Ms. Fitzgerald (participant). Ms. Fitzgerald was a Caucasian female, approximately 35 years old. She had completed a traditional teacher preparation program in elementary education and was licensed to teach grades K through 6. At the time of the study, Ms. Fitzgerald had completed eight years as a classroom teacher. Although she spent one year as a third grade teacher, the rest of her teaching experience was at the fifth- and sixth-grade levels. Ms. Fitzgerald was a looping teacher, meaning that one year she would teach fifth grade and then the next year she would move to sixth grade and teach the same students before returning to teach fifth grade again.

When asked during her selection interview if intelligence was something that could change, Ms. Fitzgerald stated, "I don't think so. I think it's something that you are born with and what you learn through school is what you learn. And that is about all you have." Further, Ms. Fitzgerald distinguished between students who were naturally talented in mathematics versus those who were talented in literacy, with little room to change.

> I think that you are either literature based or you're mathematically based. . . . Well the literature-based [students] are slower learners. Like I said, they like word problems and stuff. The mathematically based [students], they can do it through computations. They can do it with manipulatives. They can usually draw diagrams. They can see it in more vast ways.

Here, Ms. Fitzgerald indicated that a person's intelligence and mathematical ability cannot be changed. In this way, her statements aligned with an entity theory, which led to her selection as a participant in this study.

Ms. Gorman (participant). Ms. Gorman was an African American female, approximately 35 years old. She had completed a traditional teacher preparation program and was licensed to teach grades K through 8. At the time of this study, Ms. Gorman had completed nine years of teaching. Although Ms. Gorman had taught at a variety of grade levels, the majority of her experience had been at the fourth- and fifth-grade levels with her most recent experience being in fifth grade.

When asked during her selection interview if intelligence was something that could change, Ms. Gorman stated, "I look at this as saying people can make mistakes, and you can learn from your mistakes, and you can change." She explained that although some students understand procedures more quickly, *all* students can gain a stronger understanding of mathematics through struggling with difficult material and exploring and explaining their thinking.

> I think that in the long run they would have a better understanding of math, they would have the critical thinking skills to think through it, whereas some of the others, you know, they could do the procedures but couldn't explain why. I mean I had some [high achieving] kids who, at the beginning of the year, “The answer is 24.” “How do you know that?” “I just know.” They knew the procedures of it, they could tell you the answer but couldn't explain. It didn't mean anything. So I was, I know this sounds horrible, but I was almost excited when they would get something incorrect because that gave me the chance to help them think through it, and it wouldn't be, “Oh it's just right because you got the right answer.”

In these statements, Ms. Gorman indicated that intelligence and mathematical ability are malleable attributes that can be shaped through effort. She also drew attention to the role of mistakes in learning. These statements provided evidence that Ms. Gorman held an incremental theory regarding these attributes, which led to her selection as a participant in this study.

Procedures

The participant selection process occurred during the first two days of the summer institute. During days three through nine, we observed Ms. Fitzgerald and Ms. Gorman interacting during selected segments of the professional development. On four occasions we observed the participants while they were engaged in collaborative problem-solving activities (i.e., immersion experiences). We also observed them during one practice-based experience, one homework review session, and one reading discussion session. During observations, we videoed their interactions and utilized the observation protocol to generate a written record of their behaviors. We conducted additional individual interviews with Ms. Fitzgerald and Ms. Gorman on days three, six, and ten of the summer institute. Table 4 provides a timeline of the data collection procedures.

Data Analysis

To analyze the data, we drew heavily on the organization and analysis procedures described by Yin (2014). We developed a case study database for each participant by organizing and compiling all data chronologically. The data included interviews, observation protocols, and videos of immersion experiences in which participants engaged in collaborative problem solving. We elected to focus on immersion experiences as these were designed to invoke struggle in mathematics by the participants. We anticipated that these opportunities for struggle would allow the behaviors potentially aligning with the two opposing implicit theories to emerge. Although immersion experiences occurred throughout the summer institute, we chose videos of immersion experiences from Day 5 and Day 8 for transcription and analysis for two reasons. First, the tasks on these particular days (see Appendix B) clearly provided both participants with opportunities to experience mathematical struggle within the relearning context. This opportunity to experience struggle was fundamental for enabling us to answer our research question. Second, on each of these days, Ms. Fitzgerald and Ms. Gorman were in a group together. In this way, the participants experienced the same group discussions and interactions during the collaborative work, which, in turn, supported a stronger analysis of the data. The video transcriptions for the Day 5 and Day 8 immersion experiences were in the form of narratives, which included both dialogue and descriptions of actions from the videos. Although video from the remaining days of the summer institute were not transcribed for analysis, data from these days in the form of completed observation protocols were analyzed.

Table 4
Timeline for Grades 5-6 Activities and Data Collection

Day	Instrument/Data Source	Participants	Activity
1	Implicit Theories Survey	All	
2	Interview Protocol 1*	8 selected participants	
3	Interview Protocol 2*	F and G	
	Video and Observation Protocol	F and G	Immersion Experience: Problem involving modeling multiplication of mixed numbers
4	Video and Observation Protocol	F and G	Immersion Experience: Problem involving partitioning fractions
5	Video* and Observation Protocol	F and G	Immersion Experience: Problem involving division of fractions and interpreting the remainder (Appendix B)
6	Interview Protocol 3*	F and G	
	Video and Observation Protocol	F and G	Homework discussion using collaborative groups
7	Video and Observation Protocol	F and G	Practice-based Experience: Making sense of student explanations of standard and non-standard algorithms for multiplying/dividing fractions/decimals
8	Video* and Observation Protocol	F and G	Immersion Experience: Solving percent problems using rectangular grids (Appendix B)
9	Video and Observation Protocol	F and G	Article Discussion
10	Interview Protocol 4*	F and G	

Note. F and G represent the two case study participants: Ms. Fitzgerald and Ms. Gorman.
* Transcribed for analysis.

To analyze the data, we employed a deductive process (Patton, 2002) in which we relied on the categories found on the observation protocol to provide an initial means for examining the data. For each category and each participant, a member of the research team analyzed the data from the video narratives, looking for ways in which the participant was responding to the activities as related to that category. For example, when coding within the protocol category of "Success of Others," a member of the research team identified an instance in which Ms. Gorman encouraged her group members to persevere in solving the problem. In response to another group member's comment about deriving a solution to the first problem, Ms. Gorman said, "Then we can figure out how to do the rest of them" (Narrative, Day 8). In this example, the behavior was noted as a way in which Ms. Gorman interacted during the professional development activities and as evidence aligned with an incremental theory. Following this initial coding, the members of the research team exchanged categories for the purpose of verifying the data that had been coded as well as reviewing the data for additional evidence. Discrepancies were discussed by the two researchers analyzing a given category and then resolved through discussion among the entire research team.

Although this process was helpful for identifying evidence of ways in which the participants interacted during professional learning activities, we found the categories within the protocol unhelpful for identifying emerging themes. However, as we began to categorize the codes chronologically, we noted that themes began to emerge within three phases of the immersion experience: initial exposure to a task, exploration of a task, and resolution of a task. For the most part, these phases corresponded with those presented in the *Thinking Through a Lesson Protocol* (Smith, Bill, & Hughes, 2008).

The first phase, initial exposure to a task, represented the participants' initial engagement with the task, as they began to make sense of the problem. During the exploration of a task phase, participants explored the mathematics in order to gain a deeper understanding and potentially develop a solution to the problem. Finally, the resolution of a task phase began as the class came together as a large group, allowing for presentation and discussion of the small groups' solution processes. Within these three phases of the immersion experiences, patterns emerged with regard to the participants' engagement with peers, engagement with the task, and engagement with the mathematical content. As these patterns emerged, we identified and discussed data pieces from the interviews and observation protocols that provided further evidence or additional insights into the themes. Following discussions, we returned to the data, aiming to develop deeper understandings of the engagement patterns. We continued this process until we felt we had reached "saturation; that is the point in the research when all the concepts are well defined and explained" (Corbin & Strauss, 2008, p. 145). Figure 1 provides an overview of the final coding structure.

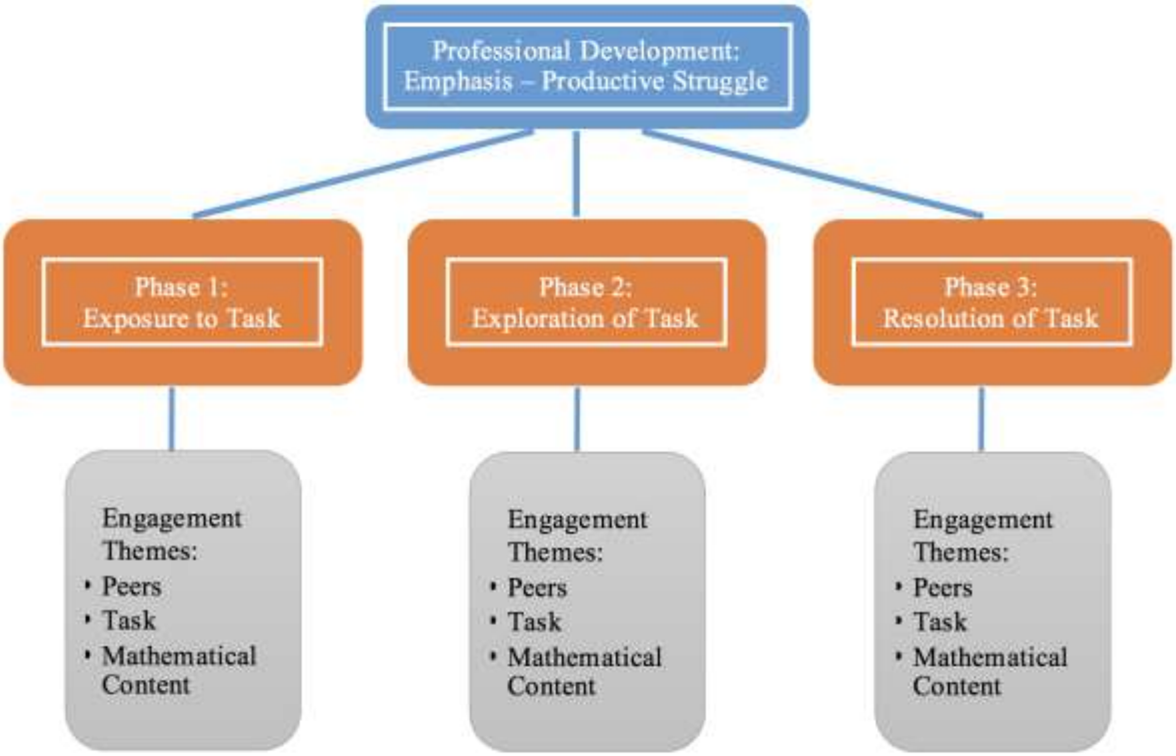

Figure 1. Representation of the coding structure.

Limitations, Delimitations, and Trustworthiness

As with any study, there were factors that affected the scope of the study that need to be stated. Factors that were not within the control of the researchers are considered limitations for the study. Limitations included the lack of males and minorities that participated in the project. In contrast, factors such as the context of the study and the decision to focus on two fifth-grade teachers were decisions made by the researchers and, therefore, represented delimitations of the study. Our intent was to develop analytic generalizations, which Yin (2014) defined as “a carefully posed theoretical statement, theory, or theoretical proposition. The generalization can take the form of a lesson learned, working hypothesis, or other principle that is believed to be applicable to other situations” (p. 68). Further, the trustworthiness of our findings is supported by the use of multiple sources of evidence, replication logic, and a case study protocol.

Presentation of Results

In the following sections, each case will be presented and organized according to the three phases of the immersion experiences. Within each phase, the discussion will focus on three aspects of engagement: engagement with peers, engagement with the task, and engagement with mathematical content. In instances where no data was coded within an engagement aspect, the aspect will be omitted from the phase. Following the two cases, a cross-case analysis is provided. Connections between implicit beliefs and the results will then follow in the discussion.

Ms. Fitzgerald

Lesson Phase: Initial Exposure to a Task

Engagement with peers. When given a task, Ms. Fitzgerald began by independently setting up the problem and, most often, employing an algorithm to obtain an initial solution. Typically, after approximately one minute of individual work, Ms. Fitzgerald examined the work of other group members and occasionally copied their solutions onto her task sheet. She told her group members, "[I'm] copying yours" (Narrative, Day 5). Often, her engagement with her peers was limited to a brief interjection of her solution to the problem without discussion of her process. For example, "I got one and a third" (Narrative, Day 5).

Engagement with a task. Ms. Fitzgerald's initial engagement with the task focused on identifying a solution and seeking confirmation from peers. During initial discussions, Ms. Fitzgerald offered short responses, seeking immediate answers from the group. Examples of such responses included: "It is one and a third" (Narrative, Day 5), and "That's what I thought, but I think 43 and a half" (Narrative, Day 8). If the group did not confirm her solution, her engagement with the task became brief and sporadic (Observation, Day 6).

Engagement with mathematical content. On the first day of the summer institute, the facilitators instructed the teachers to "throw all algorithms away" in order to focus on models or alternative representations. Despite these directions, Ms. Fitzgerald frequently chose to start with an algorithm. "I want to do the algorithm and not do the drawing. I'm the kid that will do that algorithm and make the picture to match the solution" (Interview, Day 3). Ms. Fitzgerald's choice to begin with an algorithm served as an obstacle to her engagement in the intended mathematical content (i.e., developing models of the mathematics).

When the group discussion moved towards modeling the mathematics, Ms. Fitzgerald quickly disengaged. "I don't understand it with the [models]. I don't understand it with the area model. Bar models. Any models. I can't read the models. I can't draw the models" (Interview, Day 6). Ms. Fitzgerald did not feel that she was able to draw a model to represent and solve a problem without the use of the algorithm. "I've tried – like our problem sets, you have to draw the model, no algorithm and I just look at it and go – oh, I'm defeated again today, I can't do my homework" (Interview, Day 6). At times, Ms. Fitzgerald would not share her ideas with the group. "Like I said, when I work in the groups, it's like I'm not really part of this group because I can't do the models" (Interview, Day 6).

Lesson Phase: Exploration of a Task

Engagement with peers. As Ms. Fitzgerald began further exploration of the tasks, her engagement with peers was limited to short, clarification questions such as, "So what exactly are we supposed to answer?" (Narrative, Day 5), and brief statements such as, "Yes," (Narrative, Day 8) and "Got ya" (Narrative, Day 5). In addition, Ms. Fitzgerald made comments such as, "I'm lost" (Narrative, Day 5), or "I don't even know where to start" (Observation, Day 5).

As this phase of the lesson progressed, Ms. Fitzgerald, for the most part, discontinued her engagement with her peers as evidenced by group dialogue in which she did not participate. In addition, evidence of discontinued engagement was found within statements from the video narratives, such as, “During this exchange between [the other group member] and Gorman, Fitzgerald occasionally glances to [the other group member’s] notes. Fitzgerald appears to be darkening the outline of her model, rather than adding new information” (Narrative, Day 5).

Engagement with task. While other group members discussed the task, Ms. Fitzgerald tended to disengage from the task. She took frequent breaks to leave the room for periods of time (Observation, Day 7). In addition, she often appeared to be distracted, focusing her attention on conversations unrelated to the mathematical concepts (Narrative, Day 5).

At moments when Ms. Fitzgerald engaged in the task, she frequently focused on the solution or the expectation of a task. As an example, in the following dialogue, the group had been given the answer to the problem (1 ½ servings) and asked to explain why this was the answer.

> Fitzgerald: [Gesturing lightly with right hand] So why wouldn’t she have ate [*sic*] one and a third cup, one and a third servings?
>
> Other group member: That’s the answer [points at 1 ½ servings on the handout], but that’s what I got.
>
> Gorman: [To both] So it will be what?
>
> Fitzgerald: One and a third; it is one and a third.
>
> Huntley: It’s not [pointing to the handout]. It’s one and a half.
>
> Fitzgerald: I mean one and a half. Why wouldn’t it be one and a third? Because, if one serving is two is two thirds, and she ate a cup, that’s one and a third. One serving and a third of another. . . . So what exactly are we supposed to answer? (Narrative, Day 5)

In this exchange, Ms. Fitzgerald believed the answer to be 1 1/3 and continued to think so after her group members had pointed to the answer that was given in the problem. Although she appeared to engage with the task as she pondered why her answer was incorrect, the narrative notes soon demonstrated her disengagement, with statements such as “Fitzgerald shuffles through her notes” (Narrative, Day 5), and “Fitzgerald is distracted by her phone during this exchange” (Narrative, Day 5).

Engagement with mathematical content. Ms. Fitzgerald tended to view the process of drawing models to represent mathematical ideas as an isolated procedure to be learned. This notion was revealed in an interview in which she was asked about teaching the models to her own students.

> I would let [my students] do their own models first but we might go to an algorithm a little bit quicker with them and try to work backwards if they can’t get the models on

> their own. Solve the problem [first using an algorithm] and then try to create a model that matches and then hopefully later on they could go to the model first. (Interview, Day 10)

Ms. Fitzgerald also indicated that students had a tendency to disengage when faced with challenging material. As a result, Ms. Fitzgerald would “do a whole group lesson to help lead them to the answer” (Interview, Day 3).

The ideas described by Ms. Fitzgerald in relation to teaching seemed to align with her personal engagement with the mathematical content and fixed mindset. Her limited engagement during the set up phase and frequent breaks often hindered her immersion with the mathematics during the exploration phase.

Lesson Phase: Resolution of a Task

Engagement with peers. As groups neared the resolution of a task, teachers presented their solution methods to the class, either in the form of verbal presentations or posters. At these times, teachers were expected to discuss the presented ideas with their peers. During verbal presentations and discussions, Ms. Fitzgerald rarely interacted with peers. At other times, groups circulated the room commenting on each other’s solutions as represented on posters. In these instances, Ms. Fitzgerald again chose to disengage, either passing each poster quickly or not considering the other groups’ solutions (Observation, Day 4). Sometimes, though, she commented and then left when the poster’s creator joined the group to describe the contents of the poster (Observation, Day 5), thus limiting her interaction with peers in this situation.

Engagement with mathematical content. During the resolution phase, the facilitators utilized the teachers’ presentations and peer discussions as a means for engaging teachers in the mathematical content. Ms. Fitzgerald frequently limited her attention to others’ thinking. She justified her choice to not pay attention to other groups’ presentations by saying, “I’m going to confuse myself if I try to figure it out on another one” (Narrative, Day 8). In doing so, she also limited her engagement with the mathematical content.

Mrs. Gorman

Lesson Phase: Initial Exposure to a Task

Engagement with peers. When given a task, Ms. Gorman immediately engaged with her peers, as she aimed to gain an overall sense of the problem (Narrative, Day 5). After quietly reading the problem, she asked her group members about their thoughts (Observation, Day 3). The following example is a conversation between another group member and Ms. Gorman after receiving their task.

> Gorman: [To the other group member] Was it this? Or was it the other way around? [Referring to previous problem]
> Other group member: [Nodding] Six divided by one-half.
> Gorman: So it was that?
> Other group member: Yes.

> Gorman: Ok. (Narrative, Day 5)

These purposeful questions continued back and forth, as the two group members gathered information as well as expressed their ideas (Narrative, Day 5). In addition, Ms. Gorman asked her group members questions, seeking confirmation, such as, “You could use tenths, right?” (Observation, Day 4), and “Did you go ahead and split this into thirds?” (Narrative, Day 5).

Ms. Gorman initiated most of the questioning with her peers (Observation, Day 7), yet was willing to answer questions that were asked of her (Narrative, Day 8). When she did not know the answer, she immediately began working to find one, both independently and collaboratively, telling her peer, “Let’s talk about it” (Observation, Day 6).

Engagement with task. Ms. Gorman used information from previous problems in order to help the group move forward with the task (Observation, Day 8). “So now, this is what we were talking about earlier if you think about it. Remember how we were, on our homework?” (Narrative, Day 5). At times, however, Ms. Gorman expressed her insecurity with the task by, for example, throwing her pencil on the desk and stating, “I don’t think I can do this” (Observation, Day 6). At this point, she then relied on her group members for support (Observation, Day 6).

Engagement with mathematical content. Ms. Gorman attempted to make sense of a task through a model first, although she admitted that she wanted to use an algorithm. “This [was] the way I was taught, so I think algorithm. This is how I think first” (Interview, Day 3). She acknowledged the role of drawing models in supporting the development of mathematical understanding.

> So for me it is, I guess it is just going back to the basics of just being able to draw a picture of whatever it is they are saying. So my plan this week is just to continue with that. Whatever [the facilitators] give me, to be able to create the model and try to create the model in different ways. (Interview, Day 6)

Ms. Gorman believed that models “would become one of the tools that they could use to figure out the problem” (Interview, Day 6), make sense of the mathematics, and make connections. Ms. Gorman recognized that models were a personal challenge to overcome. “I want to be able to not just draw [the model] but for the kids to be able to use the different manipulatives to prove whatever it is that they are saying” (Interview, Day 6).

Lesson Phase: Exploration of a Task

Engagement with peers. During task exploration, Ms. Gorman continued to ask questions and collaborate with her peers (Narrative, Day 5). The following exchange is provided as an example.

> Gorman: Are we allowed to go in and change? (points with her pencil onto her neighbor’s paper) So they want us to use this diagram, right?
>
> *Fitzgerald raises her head and looks over at what Gorman is explaining to the other group members.*

> Gorman: Can we go in and flip this? (pause) Even though it's shaded like this, can we move the shading?
> Other group member: Why would that matter?
> Gorman: Well I was saying (pause).
> Other group member: You still have 34 and 80 (pause).
> Gorman: Well right (pause) but at least you could say (pause) that we know it's less than (pause) half. (Narrative, Day 8)

Here, Gorman worked with the aid of her group member to explore possibilities for representing the problem as well as to make sense of the mathematics.

Engagement with task. Ms. Gorman was rarely distracted by her surroundings (Observation, Day 3). Focusing on the problem, she allowed herself to struggle the same way she wanted her students to struggle.

> Let [the students] struggle, you know, to a degree. Because you don't want them to struggle so much that they're like [done]. But to let them struggle and to see if they can figure out [the problem]; a way of persevering through [solving the problem]. (Interview, Day 10)

As she explored the task, Ms. Gorman persevered when faced with challenges (Interview, Day 6). She described herself as being overwhelmed, as her lack of understanding had been exposed.

> I'm very overwhelmed. Just in the sense of, obviously I know how to divide and multiply fractions, but having to show pictorially how to multiply and divide fractions is totally different. Because I never have really had to do that. (Interview, Day 6)

Engagement with mathematical content. Ms. Gorman was not satisfied with obtaining an answer but rather aimed for making connections and deepening her understandings. For example, when Ms. Gorman's group was working on the task from Day 8 (Narrative), they believed that they had the correct solution. Members of the group celebrated, throwing their hands up in the air, seeming satisfied with their accomplishment. Ms. Gorman leaned back in her chair with a smile on her face, giggled, and then quickly leaned toward the group. "Ok wait. So say that again" (Narrative, Day 8). The group then continued looking for connections among the mathematical representations (Observation, Day 8).

During an interview, Ms. Gorman provided insight into why she sought to develop a deep understanding of the mathematics.

> I just needed to make sure I can do it. And not just working out a problem. I need to be able to show a picture with it. But like I said – I want to be able to take it back to a classroom. (Interview, Day 6)

It seemed that Ms. Gorman believed that the type of understanding represented in connecting models with the mathematics was necessary for effective teaching. Further, Ms. Gorman described the need to understand all of the different models so that she, as well as her students, would know various methods to solve a problem.

> [I want to] make sure the kids kind of work them out in different ways so that they don't get stuck in one strategy because I think that's kind of what happened to me, was I was stuck in one strategy. (Interview, Day 10)

Lesson Phase: Resolution of a Task

Engagement with peers. While groups presented their ideas to the class, Ms. Gorman often disengaged from the resolution process and other activities, as she continued to engage with her own task (Observation, Day 5). In some instances, Ms. Gorman was so focused on working through a particular task that she failed to notice that other groups were presenting (Observation, Day 7). During other times, she whispered questions to her peers, and casually looked up at the presenters, nodded her head, and immediately returned to the task (Narrative, Day 5). While working on one particular task, Ms. Gorman became so involved with her work that she did not acknowledge her name being called by the facilitator (Narrative, Day 5). When the facilitator got her attention, Ms. Gorman shook her head and said, "Oh no. She's explaining, like that's what we're trying to figure out" (Narrative, Day 5). Rather than actively participate in the whole-class discussion, Ms. Gorman worked to make sense of the presented solution in her own way.

Engagement with mathematical content. When Ms. Gorman focused her attention on the presenters, she placed her problem to the side and began to work through the presenters' problem (Observation, Day 9). Following along with the presenters' explanations, she applied this new information to her original problem (Narrative, Day 8). Ms. Gorman also compared newly presented solution methods to those of previous presenters, producing a more organized representation (Observation, Day 6).

Similarly, Ms. Gorman attended to multiple solutions displayed on chart paper around the room (Observation, Day 3). While teachers walked in a gallery-style manner, Ms. Gorman observed how other groups solved their problems (Observation, Day 3). She found herself "trying to work out some of [the] problems in a different way than what [she] was used to" (Interview, Day 10). Ms. Gorman reflected on how this made her feel.

> And then another thing that was overwhelming is that you are passing these posters around and you are looking and somebody like me whose mind is just constantly going and I see a different way to work out something then I'm like – ooh! You know, I want to work it out this way. And then there's another way. And then another. And I'm like, you know. So then, I'm like – which way do I like the best? But just because I like it doesn't mean the kids will like it that way so then I'm back to here are all these options. And so again – overwhelming. (Interview, Day 6)

On these occasions in which Ms. Gorman attempted to make sense of others' thinking, she focused on connecting the ideas to her own understanding, thus engaging in the mathematical content in a meaningful way. This process, however, often led Ms. Gorman to ask for additional time to assimilate the others' ideas (Observation, Day 6).

Cross-case Analysis

Table 5 provides an overview of the analysis results and is intended to aid in understanding the cross-case analysis. These results are organized according to the lesson phases.

Lesson Phase: Initial Exposure to a Task

During initial exposure to a task, both participants engaged with their peers as a means for seeking confirmation of their ideas. Beyond this, however, their engagement diverged. For Ms. Fitzgerald, this initial time involved applying previously learned algorithms and asking peers to verify her answers. In contrast, Ms. Gorman sought to understand the task, posing questions to her peers and relying on her sense-making efforts to make connections to previously solved problems. In addition, Ms. Gorman aimed to understand the task via a model, a process for which she expressed her insecurity. Despite this feeling of insecurity, Ms. Gorman worked with group members to develop an initial model for the problem, while Ms. Fitzgerald, who also expressed her insecurity regarding models, began to withdraw from the group's activities.

Lesson Phase: Exploration of a Task

During the exploration phase, the patterns of engagement previously exhibited by Ms. Fitzgerald and Ms. Gorman persisted, each growing more profound. That is to say, Ms. Fitzgerald's patterns of seeking confirmation and disengaging from the work became more prominent, as did Ms. Gorman's efforts to understand the problem and persevere through moments of struggle. Like Ms. Gorman had done previously, Ms. Fitzgerald expressed her insecurity with the process of developing the models. Unlike Ms. Gorman though, Ms. Fitzgerald did not allow herself to struggle, resulting in a failure to immerse herself in the mathematics.

Lesson Phase: Resolution of a Task

As the facilitators led the resolution phase, Ms. Fitzgerald and Ms. Gorman continued their patters of engagement; however, this resulted in both teachers being *dis*engaged with the group presentations, but for very different reasons. Ms. Fitzgerald clearly stated that considering others' processes would cause her confusion; thus, she chose not to participate. Although Ms. Gorman also chose not to participate, her motivation was quite different; she demonstrated her desire to continue better understanding the problem with which she had been working. At times, however, Ms. Gorman placed her work aside and began actively considering the work of others. In this way, she was heavily immersed in the mathematics.

Table 5
Cross-case Comparison

Aspect of Engagement	Fitzgerald	Gorman
	Lesson Phase: Setting up a Task	
Engagement with Peers	• Examined and sometimes copied peers' work • Stated her answer	• Described her own understandings • Asked purposeful questions to further understanding • Sought confirmation of initial ideas • Worked independently and collaboratively
Engagement with Task	• Sought confirmation for solution • Engaged briefly and sporadically	• Tried to gain insight by connecting to prior tasks • Expressed insecurity
Engagement with Mathematical Content	• Applied algorithms • Disengaged in discussion of models	• Attempted to make sense of the task through a model
	Lesson Phase: Exploration of a Task	
Engagement with Peers	• Provided clarification questions and brief statements • Commented on inability to solve problems • Discontinued engagement with peers	• Continued to ask purposeful questions • Collaborated with peers
Engagement with Task	• Took frequent breaks • Seemed distracted • Focused on solutions and expectations	• Not distracted by others • Persevered in solving problems • Allowed herself to struggle
Engagement with Mathematical Content	• Indicated that each model is an isolated procedure to be learned • Hindered immersion into the mathematics	• Sought understanding, was not satisfied with an answer alone • Looked for connections among mathematical representations
	Lesson Phase: Resolution of a Task	
Engagement with Peers	• Rarely interacted with peers	• Disengaged from the resolution process at times
Engagement with Mathematical Content	• Limited her attention to others' thinking	• Attempted to make sense of others' thinking in terms of her own understanding • Needed more time to assimilate others' ideas

Discussion and Conclusion

Supporting students in learning mathematics is a daunting task that requires teachers to develop a deep understanding of the mathematics being taught (Ball, Thames, & Phelps, 2008). Professional development represents the primary means to support teachers in developing this knowledge (Loucks-Horsley et al., 2010). Recognition should be given, though, that teachers are *re*learning mathematics and that their prior knowledge often represents incomplete or incorrect understandings (Zazkis, 2011). Therefore, teachers must participate in transformative learning (Smith, 2001) and experience productive struggle (NCTM, 2014) as they engage in immersion experiences (Loucks-Horsley et al., 2010). The role of implicit theories in these experiences had not been previously examined.

Therefore, the purpose of this study was to examine how teachers ascribing to varying implicit theories engaged in activities associated with a professional development institute. We selected Ms. Fitzgerald and Ms. Gorman as our participants as they represented different implicit theories. Our analysis provided valuable insight into our research question: How are the implicit theories held by elementary mathematics teachers related to patterns of engagement in a professional development setting designed to invoke productive struggle during the relearning process, if at all?

As Ms. Fitzgerald and Ms. Gorman participated in this professional development, patterns of engagement emerged as they faced incidences of struggle with the mathematical content. As the participants moved through the three lesson phases, these patterns became more prominent. Further, these patterns remained consistent across the varying days of the summer institute. As a consequence, the participants' engagement heavily influenced their opportunities to participate in productive struggle during this relearning context. For both participants, the patterns of engagement seemed to be closely related to their implicit beliefs.

Initially, Ms. Fitzgerald applied an algorithm and sought confirmation of her answer, demonstrating her performance-goal orientation as she sought validation of her ability to solve the problem. Once confronted with a moment of struggle, however, Ms. Fitzgerald proclaimed that she was not able to develop the required models. Her continued insistence that she was not capable of developing models suggested that she viewed this as a fixed attribute that she could not improve. As a result, her inclination was to avoid the challenge by taking frequent breaks, appearing distracted, and disengaging from discussions. By disengaging, she avoided receiving perceived negative judgment regarding her ability. These actions also lessened the opportunity for Ms. Fitzgerald to produce new learning when she encountered mistakes, as she did not engage in a way to allow for mistakes (Boaler, 2016). Ms. Fitzgerald's propensities to seek validation regarding her ability, to view her ability as fixed, and to avoid challenge, thus avoiding critique, were aligned with an entity theory (Dweck, 1986; Dweck & Leggett, 1988; Elliott & Dweck, 1988). These propensities associated with the entity theory likely did not effectively support Ms. Fitzgerald's engagement in this relearning context.

In contrast, Ms. Gorman seemed interested in expanding her knowledge as she studied a variety of models and made connections among them. Her aim of improving her ability demonstrated her learning goal orientation. When faced with struggle, Ms. Gorman at times indicated that she was not able to overcome the struggle; yet, she continued working and allowed herself to struggle. Ms. Gorman's acceptance of struggle in the relearning process provided opportunities for her to encounter and attend to mistakes in ways that supported her engagement in productive struggle and likely enabled deeper learning. Thus, she demonstrated her view of ability as malleable. As she engaged in the task, she pursued the challenges present and persisted in her effort to achieve her goals. Ms. Gorman's tendencies to seek to expand her knowledge, to allow herself to struggle, to view ability as malleable, and to pursue and persist with challenges, aligned with an incremental theory (Dweck, 1986; Dweck & Leggett, 1988; Elliott & Dweck, 1988). These tendencies associated with an incremental theory appeared to effectively support Ms. Gorman's engagement in this relearning context.

Despite these contrasting experiences, one common engagement pattern emerged that initially seemed to stifle both participants' opportunity for relearning, as they entered the resolution phase, neither participant fully engaged in the whole group discussion. Ms. Fitzgerald's lack of full engagement was potentially expected, as her pattern of disengagement had been established. For Ms. Gorman, however, the failure to fully engage in the resolution process was not expected, as it could have provided an opportunity to expand and deepen her understanding. Her learning goal orientation seemed to explain this disengagement, as she sought to master the content with which she was working prior to engaging in the whole group discussion. Once ready to engage, though, she did so in a meaningful way.

An additional contrasting pattern of engagement emerged that influenced the participants' experiences yet did not seem to be related to the participants' implicit beliefs. Within the exploration phase, Ms. Fitzgerald indicated that the models were an additional procedure to be learned and did not attempt to make connections between the models and the algorithms. In contrast, Ms. Gorman looked for connections among the different models and seemed to understand the support the models provided for understanding the algorithm. When reflecting on this difference, one might argue that Ms. Fitzgerald's view may have been espoused to avoid the challenge of making the connections. No clear evidence was found, though, that supported this claim, and further investigation is needed. Still, this contrasting pattern of engagement clearly influenced the participants' engagement with the mathematical content and, in doing so, influenced their relearning experiences.

From these findings, we offer three implications. First, the impetus for this study was to examine how the patterns of engagement during the relearning process might align with the implicit theories model (Dweck & Leggett, 1988). Our results demonstrated that both Ms. Fitzgerald's and Ms. Gorman's engagement with peers, tasks, and mathematical content aligned with the theory. Therefore, a theoretical implication is the potential extension of the implicit theories model into the relearning context. It is important to note, though, that there are limitations of applying the implicit theories model to a complex social context such as professional development. Therefore, future research is warranted that not only involves more participants but also investigates alternative explanations for the engagement patterns (e.g., beliefs regarding the nature of mathematics).

Second, the professional development that provided the context for this study touted a research-based design, focusing on a transformative experience (Smith, 2001) via a combination of practice-based and immersion experiences (Loucks-Horslely et al., 2010). Despite this alignment with the literature on effective professional development, the patterns of engagement for Ms. Fitzgerald and Ms. Gorman were different at the onset of the professional development and remained so over the course of the 10 days. There was no evidence that the professional development itself influenced the implicit beliefs of the participants or altered the patterns of engagement. For Ms. Fitzgerald, this likely led to an ineffective environment for relearning. Therefore, our results suggested the need for purposeful attention to the role of implicit theories in relearning during professional development. Although much of the current practitioner literature focuses on the development of a growth mindset (i.e., incremental theory) in mathematics students (e.g., NCTM, 2014), future work should examine whether such a focus on implicit beliefs and students is sufficient for supporting all teachers' engagement in the relearning context. If not, purposeful attention will need to focus on the effects of implicit beliefs in a relearning context, a teacher's implicit beliefs, and the teacher as the learner of mathematical content.

Finally, although not necessarily connected to the implicit theories, our findings suggested that the role of mathematical representations and models should be continuously revisited during professional development, so that teachers are better prepared to engage in the relearning process. Ms. Gorman saw the models as a means for deepening students' mathematical understandings and as a necessary component for understanding algorithms. This seemed to support her choice to establish drawing models as a learning goal for herself. In contrast, Ms. Fitzgerald indicated that models and their corresponding algorithms were different procedures to be learned. Therefore, Ms. Fitzgerald did not acknowledge the importance of developing the ability to draw models and chose to disengage from this process. Perhaps holding a better understanding of the role of models in the mathematical learning process could serve as a stimulus for engaging in the relearning process, regardless of the implicit theories a teacher holds. Future work should examine this possibility.

From our results, we find it is critical to attend to the implicit theories held by mathematics teachers as they engage in the relearning process. Attention to these theories will serve to enhance teachers' relearning experience and, in turn, will help them develop a deeper understanding of the mathematics they teach. Such understanding is a critical component of effective teaching and serves to support the mathematical success of students.

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Appendix A

Overview of Mathematical Topics

	Grade Band		
Day	K-2	3-4	5-6
1	Routine vs. Non-Routine Problem Solving	Introduction to Fractions	Models for Addition and Subtraction of Fractions
2	Classifying Numbers by Use and Part-Whole Diagrams	Multiple Meanings of Fractions	Models for Multiplication of Fractions
3	Composition and Decomposition of Numbers	Fraction Equivalence	Multiplication Properties and Fractions
4	The Equal Sign and Addition/Subtraction Problem Types	Unit Fractions and Comparing	Models for Division of Fractions
5	Number Sense and Operations	Ordering Fractions and Mixed Numbers	Models for Division of Fractions (continued)
6	Student-generated Algorithms for Addition and Subtraction	Addition and Subtraction of Fractions	Models for Division of Fractions (continued)
7	Equal Sharing Problems	Models for Multiplication of Fractions	Making Sense of Division Algorithms
8	Unit Fractions	Multiplication Algorithm for Fractions	Ratios and Proportions
9	Non-Unit Fractions	Interpreting and Comparing Decimals	Ratios and Proportions (continued)
10	Fraction Strips	Fractions and Decimals	Ratios and Proportions (continued)

Appendix B

Sample Tasks

Day 5 – Tonya and Chrissy Task (Beckmann, 2008, p. 212)

Tonya and Chrissy are trying to divide 1 ÷ $\frac{2}{3}$. They decide to consider the following problem:

One serving of rice is $\frac{2}{3}$ *of a cup. I ate 1 cup of rice. How many servings of rice did I eat?*

To solve the problem, Tonya and Chrissy draw a square divided into three equal pieces, and they shade two of the pieces.

Tonya says, "There is one $\frac{2}{3}$ cup serving of rice in 1 cup and there is $\frac{1}{3}$ cup of rice left over, so the answer should be $1\frac{1}{3}$.

Chrissy says, "The part left over is $\frac{1}{3}$ cup of rice, but the answer is supposed to be $\frac{3}{2}$ or $1\frac{1}{2}$. Did we do something wrong?"

Help Tonya and Chrissy.

Day 8 – Set A from, The Case of Randy Harris (Smith, Silver, & Stein, 2005, p. 12)

Description: Eight rectangular grids are provided. Each grid has a portion shaded. The directions ask for the fraction, decimal, and percent represented by the shaded region.

a. 8 x 10 grid with 16 squares shaded
b. 10 x 8 grid with 34 squares shaded
c. 8 x 8 grid with 48 squares shaded
d. 5 x 4 grid with 9 squares shaded
e. 10 x 10 grid with 46 squares shaded
f. 8 x 9 grid with 18 squares shaded
g. 8 x 10 grid with 30 squares shaded
h. 5 x 10 grid with 27 squares shaded