Centroid and Theoretical Rotation: Justification for Their Use in Q Methodology Research

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This manuscript's purpose is to introduce Q as a methodology before providing clarification about the preferred factor analytical choices of centroid and theoretical (hand) rotation. Stephenson, the creator of Q, designated that only these choices allowed for scientific exploration of subjectivity while not violating assumptions associated with other choices like principal components (PCA) and Varimax. Although Q software offers Stephenson's preferred choices as factor analytic options, today most Q methodologists use the more "modern" factor analytical choices of PCA and Varimax. Similarly, reviewers and critics of Q research often question the use of centroid with theoretical rotation, further discouraging their use. Researchers who took statistics coursework since the advent of statistical computer software are unfamiliar with centroid and theoretical rotation, their history, their processes, and why they offer a means of best scientifically exploring pragmatic, meaningful factor analytical solutions within Q methodology studies. Statistical versus theoretical considerations are discussed.

Q is a methodology for subjective science. Unlike Likert-scale type surveys that most often present views in aggregate within a portrait of a singular view, Q methodology is specifically designed to reveal and describe the multiple perspectives within a group of people (Brown, 1980; McKeown & Thomas, 1988; Stephenson, 1953). Also unlike typical Likert-scale type surveys, Q methodology (Q) uses a mixture of qualitative and quantitative methods, procedures, and philosophies (Ramlo, 2015). The procedure of Q possesses the following stages: development of the concourse of communications (statements); the selection of the Q-sample (a subset of the concourse); the sorting of the Q-sample (Q sort); analysis of the sorts which involves correlation and factor analysis which results in the determining of Q factors; interpretation of those factors; and their description as perspectives (Brown, 1980; McKeown & Thomas, 1988; Newman & Ramlo, 2010). Within this manuscript, the purpose is to discuss and promote specific choices within the factor analytic stage of Q methodology (centroid extraction and theoretical rotation) and, therefore, the discussion will focus on this singular aspect of the larger methodology while including philosophical, ontological, and epistemological considerations related to this stage.

The Purpose of Factor Analysis in Q

Within Q methodology, the factor analysis' purpose is to group similar views into factors where the views are captured in the form of Q sorts (Brown, 1980; Ramlo & Newman, 2011; Stephenson, 1961). These views, whether multiple views offered in a single-case study or by multiple individuals, can each be assumed unique until the factor analysis is performed (Stephenson, 1961). If the participants' views were idiosyncratic, then each sorter would have his own factor—resulting in a factor for each person who sorted (Brown, 1978). The sophisticated statistics often reveal subjective patterns that could be overlooked by the most judicious qualitative researcher (Brown, 2008). Conceptually this revealing of patterns is not very different from more common factor analysis such as R factor analysis. Yet it is worthwhile to provide a brief review of factor analysis in general before specifically discussing factor analysis in Q in greater detail.

A Brief Summary of Factor Analysis

Factor analysis is one of a variety of different data reduction techniques. Factor analysis involves correlation and results in the grouping of similar items (in R factor analysis) or of similar views (as expressed by individuals' Q sorts). Although some may state that Q methodology consists simply of the rotated R matrix, this represents a misunderstanding of the factor matrix in Q methodology and the source of its data (Brown, 1980). Data reduction techniques are important because they may broaden the researchers' ability to interpret the data in a more effective manner (Newman & Ramlo, 2010; Stephenson, 1961; Thomas & McKeown, 1988). In essence, data reduction seeks to reduce the amount of data without the loss of a significant amount of information (Newman & Ramlo, 2010).

The SAGE Dictionary of Social Research Methods (Jupp, 2006, p. 114) contains this definition of factor analysis: "A set of procedures used to simplify complex sets of quantitative data by analyzing the correlations between variables to reveal the small number of factors which can explain the correlations." Factor analysis groups variables empirically, meaning that the groups represent correlations among the test-items, in R factor analysis, and among the individual's views (Q sorts) in Q methodology (Newman & Ramlo, 2010; Ramlo & Newman, 2011). How the groups emerge represents the factor "structure" which consists of the numbers of factors that emerge and the factor loadings (correlations) of the individual items (or sorts) on those factors (Jupp, 2006; Stevens, 2002). Factor loadings represent the correlation of a variable to a factor (Stevens, 2002).

During the factor analysis process, factors are said to be "extracted" from the data matrix and, in R factor analysis, most common methods consist of component analysis or principal components analysis[1], often referred to as PCA (Jupp, 2006; Stevens, 2002). PCA is the default extraction method in many popular statistical packages such as SPSS, contributing to its popularity. PCA assumes that each item is invariant (correlated at 1.00 with itself as represented by the use of 1's in the diagonal) (Newman & Ramlo, 2010). In PCA the first principal component accounts for as much of the variability in the data as possible (Ho, 2006).

Factor rotation typically follows factor extraction. In R factor analysis rotation is required so the factors are interpretable (Stevens, 2002). The rotation "sharpens" the factor structure and provides simple structure. In R factor analysis, the most common factor rotation method is Varimax and it is generally accepted as the best rotation method for producing simple structure (Rummel, 1970). Varimax's popularity is based on its ability to mathematically provide the clearest, maximized separation of factors and, therefore, the simplest structure (Ho, 2006).

[1]Because PCA does not have communality estimates, such as R squares or reliability estimates, in the diagonal, many statisticians would argue that PCA is not a true factor analysis. The matrix diagonal has 1s in PCA and not communality estimates. This debate about whether PCA is a type factor analysis is unimportant within the context of this paper. PCA will be referred to as a factor extraction technique throughout this paper since that is how it is treated frequently.

Varimax, the default rotation in typical statistical packages like SPSS, is an orthogonal factor rotation method. As Ho (2006) stresses for R factor analysis, simple structure is seen as a way to make the factor analytic results more meaningful although we would like to stress here, and later in this manuscript, that simple structure is mathematical, not theoretical. Here, theoretical is in a qualitative sense with connections to development of hypotheses and ideas. Such a purpose fits into Ridenour and Newman's (2008) research continuum for qualitative research. In other words, the factor analytic process in Q has a more qualitative approach than a quantitative one conceptually. Ramlo (2015) described Q's focus on theoretical significance as a qualitative approach to this mixed methodology called Q.

Abduction & Theoretical Significance

Stephenson possessed doctorates in both physics and psychology. This helped differentiate his approach to psychological research. Certainly, Stephenson's (1953) view of science was not characterized by a detached and skeptical perspective completely external to the research process, which was so typical of psychology during his day. Instead, Stephenson's view of science focused on observing, interpreting data, proposing explanations, and communicating results (Good, 2010). "Inquiry requires identification of assumptions, use of critical and logical thinking, and consideration of alternative explanations" (National Research Council, 1996, p. 23). Stephenson's methodology is based upon these scientific characteristics and his preferences for centroid and theoretical rotation at the factor analytic stage represent that same focus on observation, interpretation, and consideration of alternative explanations. Such choices for inquiry are ontological and philosophical in nature (Ramlo, 2015). The choices of PCA and Varimax when seeking factor solutions in Q methodology can provide better solutions statistically but limit the scientific process of exploring alternative explanations because of violating assumptions of a singular, best mathematical solution (Brown, 1980; Stephenson, 1953). Stephenson (1961) described abduction as an important aspect of such explorations as Q researchers attempt to provide new explanations and theories.

Abduction

Abduction is associated with inductive and deductive reasoning but is typically less familiar. American philosopher Charles Sanders Peirce introduced abductive reasoning as a logical form of guessing. More formally, abductive reasoning is the process of generating hypotheses, theories, and/or explanations (Mirza, Akhtar-Danesh, Noesgaard, Martin, & Staples, 2014). Abductive reasoning involves integration and justification of ideas to develop new knowledge (Raholm, 2010). As such, abduction is a first stage of inquiry. Saliency within the factor structure is provided through abduction in Q (Brown, 1980, 1993). Stephenson (1961, p. 7) explained that abduction, when coupled within theoretical rotation, facilitates the invention or creation of hypotheses related to subjectivity. According to Ridenour and Newman (2008), this process represents characteristics of qualitative research. However, the factors that emerge from Q's factor analytic stage are "solely the function of the Q-sorters themselves" (Brown, 2008, p. 701) and those relationships, among the sorts, remain during the factor analytical stage despite choices related to extraction and rotation. The factor analytic choices simply enable the researcher to bring into focus those relationships in terms of perspectives. Stephenson (1953) argued that the choices that best fit that exploration was centroid extraction and theoretical rotation coupled with abduction.

Stephenson and Q Methodology

Stephenson (1953) explicitly argued for the alternatives of centroid and theoretical rotation throughout his lifetime (rather than PCA with Varimax or other choices that seek mathematical simple structure). These preferences were based upon his vision for best measuring subjectivity in conjunction with his tutelage by Spearman and his knowledge of quantum mechanics (Good, 2010). Rather than an outdated stance, Stephenson's continued argument was based upon the indeterminacy of the centroid solution. Indeterminate means there is not one best solution but instead an infinite number of possible solutions. None of these infinite number of solutions violate statistical assumptions. PCA on the other hand has a single best mathematical solution. The indeterminacy of the centroid coupled with the ability to consider abductive reasoning and scientific inquiry best fits the idea of seeking operant subjectivity and offering Q as a methodology for subjective science (Brown, 1980, 1986, 1998; Stephenson, 1953).

For Stephenson, Q was unique in that it offered a means to measure the "inner experience" of those under study, which he saw as "the opposite of scientifically objective" where the term "objective" is often used to mean "as observed by others" (Stephenson, 1953, p. 23). Analyzing the connections between the factors and the sorts is a theoretical matter. In other words, through the use of factor analysis, the Q methodologist is developing theory about the different views (which one could call world views) that exist within a group within the constraints of the Q sorts (Newman & Ramlo, 2010). In this way, Q methodology is a unique way of revealing what Brown (1998, p. 3) called "the qualitative aspects of human behavior" with factors providing theoretical structure of subjective phenomena.

From an unpublished manuscript, Stephenson (1976, p. 62) explains the following:

> The sound use of Q-methodology depends on two principles of factor theory, first, that the concern is with operant factors, and second, that the fewer factors, the better, provided (usually) there are more than two. The idea of an operant factor is like B. F. Skinner's concept of operant behavior, but it is of older vintage, originating with Charles Spearman, founder of factor analysis. Operant factors are found, and, when found, self-evidently fit the interactional situation always at issue. They can only be reached satisfactorily by way of rotations of the centroid solution described in Chapter I, and by "hand rotation," i.e., graphically, not by statistical methods now much in vogue using least square or other procedures, such as a Varimax solution for simple structure. This is not because of complexities that statistics cannot assuage, but because concrete situations are always at issue, no two ever being sufficiently alike to make standardization of procedures feasible.

What Stephenson (1976) seems to be calling for here is a change in paradigm, compared to that of R-methodological studies. Within Q, he invites scientific inquiry where the context of the study is considered and explored. Theoretical rotation after centroid extraction offers a means of that scientific exploration (Stephenson, 1953).

Historical Account of Centroid and Theoretical Rotation

Although Cyril Burt[2] has also been credited for creating centroid extraction (under the name of simple summation method), most often Louis Leon Thurstone (1931, 1935, 1947) is credited with the development of the centroid method of factor extraction and Stephenson (1976) referred to centroid use in Q as "Thurstone's centroid." Before the computer age, centroid was held in high esteem because it facilitated the calculations for factor analysis (Jones, 2007). Essentially, Thurstone found the more rigorous principal factor solution infeasible with then existing computational machinery (an adding machine; Bock, 2007). The centroid solution is, put simply, the first-order mathematical approximation of the principal components solution.

Centroid Factor Extraction

It is the pre-computer age use of centroid as a more computational friendly alternative to PCA and component solutions that is most often remembered by the quantitative research community. Today, the centroid is considered obsolete and outdated (Choulakian, 2003). Thus, it is not unusual for a reviewer to respond to Q studies with comments about the use of centroid as an inappropriate option for factor extraction when PCA is easily computed within typical statistical software such as SPSS and SAS (and where centroid is not offered as an option for factor extraction).

In centroid, communality estimates (sum of the squared factor loadings) are used in the matrix diagonal instead of 1s, as in PCA. The squared factor loading is the percent of variance in that indicator variable explained by the factor (Bock, 2007; Thurstone, 1947). Therefore, in Q methodology, the communality estimates are the percentage of a person's Q sort associated with the Q sorts of the other subjects in the study (Brown, 1980). Brown (1980, p. 216) offers a detailed explanation of how these are calculated. On page 224, he also explains that

> Communality is important since during rotation the factor loadings change; however, since h^2 expresses a person's relationship to all others, which is fixed within the context of the study, the sum of the squared factor loadings should still equal h^2 no matter how changed the loadings may be.

In addition, Choulakian (2003) provides two mathematical optimality proofs for the centroid extraction. Specifically, he demonstrates that centroid use yields factor loadings with the highest sum of absolute values even in the absence of the PCA constraint that the squared component weights be equal. In other words, both PCA and centroid maximize the same variance but with different constraints. Yet, in terms of factor loadings, PCA and the centroid method maximize different functions while subjected to the same constraint. Others (Brown, 1971; McKeown & Thomas, 1988) have discussed the similarity among extraction methods previously. Similarly, Tucker and MacCallum (1997) offer comparisons of PCA and centroid with simulated data including correlation matrices and communalities. Solutions from PCA and centroid are equivalent, or nearly so, in their examples without discrepancy of fit within the data. Certainly, Q researchers have also noted similar solutions with both centroid and PCA (Brown & Robyn,

[2] Cyril Burt (1941) also created Q factor analysis. Q factor analysis also groups people but uses objective, rather than subjective data. Stephenson and Burt (1939) agreed to disagree about their two methods to group people.

2004). However, when Tucker and MacCallum (1997) used simulated data that contained discrepancies of fit, centroid and PCA offer somewhat different solutions. Yet, different or similar results with centroid and PCA do not provide the insight necessary to answer the question regarding Stephenson's specific preference of centroid for factor extraction.

The key is the centroid's indeterminateness. PCA offers one best *mathematical* solution. However, centroid does not offer one best solution (Brown, 1980; Stephenson, 1953). It is this indeterminacy that led R methodologists to abandon centroid extraction and replace it with PCA once the latter's calculation was facilitated through computer software (Bock, 2007). This is why centroid is not offered as an option in statistical software such as SPSS. Yet, it was this same indeterminacy that led Stephenson to favor centroid over PCA; without one best, mathematical solution, the researcher is able to explore the multiple best solutions using abduction (Brown, 1980; Stephenson, 1953). This exploration includes examining the extraction of different numbers of factors. In PCA, there are statistically accepted means to determine when to stop factoring including the typical use of eigenvalues of 1 or greater and scree plots. In centroid, however, the decision about how many factors to extract is a bit more "sketchy" mathematically (Tucker & MacCallum, 1997). This "sketchiness" invites the researcher to make theoretical, rather than solely mathematical, decisions about the number of factors to extract.

It is important, therefore, to understand that Stephenson's preference for centroid extraction did not represent a lack of mathematical or statistical understanding. Stephenson was a student of the creator of factor analysis, Charles Spearman, and possessed advanced degrees in physics and psychology (Good, 2010). Physics influenced Stephenson's rationale for his factor analytic preferences; his preferences were based on a desire to measure subjectivity and included quantum mechanical considerations (Brown, 1980, 1998; Stephenson, 1953).

It is also important to note that Stephenson and Thurstone (in addition to Spearman and Burt) were contemporaries. Stephenson first published an article describing Q methodology in *Nature* in 1935 just four years after Thurstone's first publication about the centroid method, "Multiple Factor Analysis," in *Psychological Review*. In that manuscript, Thurstone first presented the idea of the centroid method as an unrestricted multiple-factor model that estimated the factor loadings (Bock, 2007; Thurstone, 1931). In physics, the centroid of an object can be equated with the center of gravity; simply, the center of gravity is where an object can be balanced. Similarly, Thurstone referred to centroid factor extraction as a way of defining "centers of gravity" embedded in a correlation matrix (Widaman, 2007).

Theoretical Rotation

Theoretical rotation is often referred to as either hand rotation or judgmental rotation in the Q literature. Tucker and MacCallum (1997) refer to it as graphical rotation. Like centroid extraction, theoretical-rotation was common prior to the computer age. Whether plotted by hand or within the PQMethod software using the QROTATE option, theoretical-rotation involves plotting factor loadings for pairs of factors (e.g. Factor 1 and Factor 2) using the loadings as Cartesian coordinates. The original axes are then rotated (most easily within the QROTATE option of PQMethod, rather than using paper, pencil, and graph paper) which preserves the orientation of each sort relative to the others but, instead, changes their orientations relative to the axes (Schmolck, 2002).

Before the creation of software for theoretical rotation, the process was time intensive. Some researchers saw the use of computers as a way to quickly and easily perform factor rotations more objectively (Browne, 2001). Thompson (1962) was not worried about objectivity in relation to theoretical rotation. He questioned whether theoretical rotation would ever become outdated due to the concern about mathematically exact solutions not mirroring reality; he was not talking specifically about Q methodology but, instead, all factor analysis. Similarly, Browne (2001) stated concerns that those with little factor analysis training would accept computer-generated rotations, such as Varimax, without question. Tucker and MacCallum (1997) discuss graphical rotation in relation to Thurstone's centroid and explain that the strength in theoretical rotation is with the judgments in regard to whether the factor structure offers a meaningful solution. They remind us (in terms of R factor analysis) that the purpose of factor rotation is to provide the bases for conjecture for structure and relationships among attributes. Certainly, this argument is similar to those presented by Stephenson (1953, 1976) for use of theoretical rotation within Q studies, following centroid extraction.

Why Use Centroid and Theoretical Rotation in Q?

Brown (1980), Browne (2001), Stephenson (1953) and Thompson (1962) saw an opportunity for considerable subjective interplay by the researcher within hand-rotation and this was seen as beneficial because each decision to change the factor axes' orientation is guided by not just the data but by the researcher's knowledge regarding the variables. This is contrary to the more typical, modern day R-methodological view that the researcher should remain external to the data (Brown, 1980). Yet the research purpose connected to R methodology (to explore or confirm the factor structure of objective items such as tests) is very different from that associated with Q methodology (investigating subjectivity as represented by describing the different views about a topic). The type of data used in these two different methodologies is also different, as is already implied by their purposes. Whereas R involves objective tests, Stephenson (1953) was concerned with the subjective aspects of behavior. The underlying construct of the Q sort is that it allows participants to present their internal, subjective view (Brown, 1980; Newman & Ramlo, 2010; Stephenson, 1953). These views are replicable on the order of 0.80 as a conservative, test-retest correlation for individuals performing a Q sort (Brown, 1980).

No matter what choices are made at the factor analytical stage of Q, the interrelationships of the sorts are fixed by the sorters themselves, via their Q sorts. Rotation only serves to change the axes and does not rotate the sorts (Brown, 1980; Stephenson, 1953). The operant factors are also replicable as well as observable (Brown, 1980). Like Brown (1980) and others external to Q (Browne, 2001; Thompson, 1962; Tucker & MacCallum, 1997), Stephenson (1953) saw the benefit of researchers engaging with data through theoretical rotation in Q. Specifically, as already mentioned, the combination of centroid and theoretical rotation offers the researcher the opportunity to engage in abductive reasoning. Brown and Robyn (2004) offer a detailed account of Peirce's development of abduction as well as several examples of abductive reasoning including one by Stephenson which is also offered elsewhere in greater detail (Stephenson, 1956). Stephenson (1953) explained that centroid and theoretical rotation, coupled with abduction, not only allowed for the data reduction of sorts into factors but these choices also offer researchers the ability to perform scientific explorations such that new understandings can be discovered (Stephenson, 1961).

Thus, the goal of factor analysis within Q is to best explore the existing relationships among the Q sorts. Factor analysis provides a data reduction technique that allows the researcher to theoretically "sharpen" the structure and discover what is operant—thus the phrase operant subjectivity. This "sharpening" is not mathematical (as in simple structure) and is at the heart of statistical choices within the factor analytic stage of Q methodology (Brown, 1980). Stephenson (1953) called this "simplest structure."

This theoretical focus on "simplest structure," rather than one of statistical calculation and simple structure, is often a point of contention with those unfamiliar with or who possess misconceptions about Q methodology (such as journal editors and reviewers as well as dissertation committee members and those new to the Q community). Within manuscripts, dissertations, and professional presentations, it is often insufficient to simply state that Stephenson explicitly argued for the alternatives of centroid extraction and theoretical rotation (although this is a true statement). It is preferable to offer further elaboration from Stephenson, including his arguments related to the indeterminacy of the centroid solution and why this offers an extraction solution that does not violate assumptions (because there is not one best mathematical solution). In addition, Q methodologists should be able to present arguments related to theoretical rotation and its relationship with theoretical significance rather than statistical significance (Brown, 1980; Brown, 1986; Stephenson, 1953), while drawing on Q's philosophical, ontological, and epistemological ideas (Ramlo, 2015). Q methodology's statistical preferences are based upon the underlying purpose of the methodology to scientifically measure subjectivity (Brown, 1980, 1993, 1998; Good, 2010; Stephenson, 1953). Stephenson's (1953) purpose was to create a new, more effective approach to study human behavior and subjectivity.

Q is not R—Theoretical Rather than Statistical Considerations

Stephenson (1953) explained that Q is not simply a statistical technique. The importance of Q factors is not statistical but, instead, in their practicality. Q factors are operants in that they represent a snapshot of perspectives that are functional and pragmatic rather than representing logical distinctions (Brown, 1993). Operant is defined in psychology as an item of behavior rather than a response to a prior stimulus.

Thus, philosophically, Q has a focus on pragmatic results that offer operant subjectivities, which are offered as Q factors. This pragmatic, theoretical approach to factor analysis in Q means that Q methodologists need to move beyond statistical considerations. Such an approach embraces indeterminacy and welcomes researcher use of abductive reasoning over hypothetical-deductive or inductive reasoning. The number of factors, explained variance, and eigenvalues are not noteworthy in Q (Brown, 1978). For instance, the strength of a factor (e.g. eigenvalue and variance accounted for) is not relevant in Q. However, Brown (1980, p. 67) states that "five or six persons loaded significantly on a factor are normally sufficient to produce highly reliable factor scores, and it is in terms of the relationships among the factor scores that general statements about an attitude are made." However, there is no requirement for the number of sorters represented by a factor to make it worth consideration. In fact, in the next section, a case where a factor had theoretical significance with only one person represented is discussed (Example 2: Psych Ward).

Similarly, Q methodology does not depend on generalizability as is often important in quantitative studies. Instead, generalizability in Q is related to its ability to describe and facilitate understanding about the views for an individual or a specific group of people, whether a single-case or multi-person study, respectively. In this way, generalizations in Q have to do with types such that for those who would also highly load on (correlate with) a certain factor, the factor description is sufficient to describe their view and provide useful insight (Stephenson, 1961). A large number of persons on a factor is not required to lead to sufficient descriptions of each factor / viewpoint although, as previously mentioned here, a factor with five or six Q sorts represented provides highly reliable factor scores and relationships among the factors (Brown, 1980). In other words, factor size is not equivalent to its theoretical importance (Brown, 1978). Recall that the purpose of using Varimax, following PCA, is to find the one best statistical solution. With a statistical lens, a Q researcher could miss the importance of a factor which represents the view of only one person out of thirteen but a theoretical lens enables a Q researcher to see the importance of a factor not based on statistical outputs but based on theoretical significance, as the three examples that follow reveal.The second and third examples are summaries of previously published studies. However, the first study example has not been published elsewhere and was chosen specifically to describe details about the factor analytic choices and the importance of theoretical considerations and abduction.

Example 1: Instructor Views of Student Epistemology Relative to Student Views
Educational researchers' interests in students' epistemological views of learning and knowledge have existed for a long time (Perry, 1968). A variety of studies indicate that student epistemological views have important implications for student learning and instruction. In this study, Q methodology was used to determine the various views about learning in three different college classrooms: Technical Physics: Mechanics, Developmental Mathematics I, and Mechanical Design III. The concourse was derived from the popular Likert-scale survey developed by Schommer (1990) as well as statements taken from student interviews.

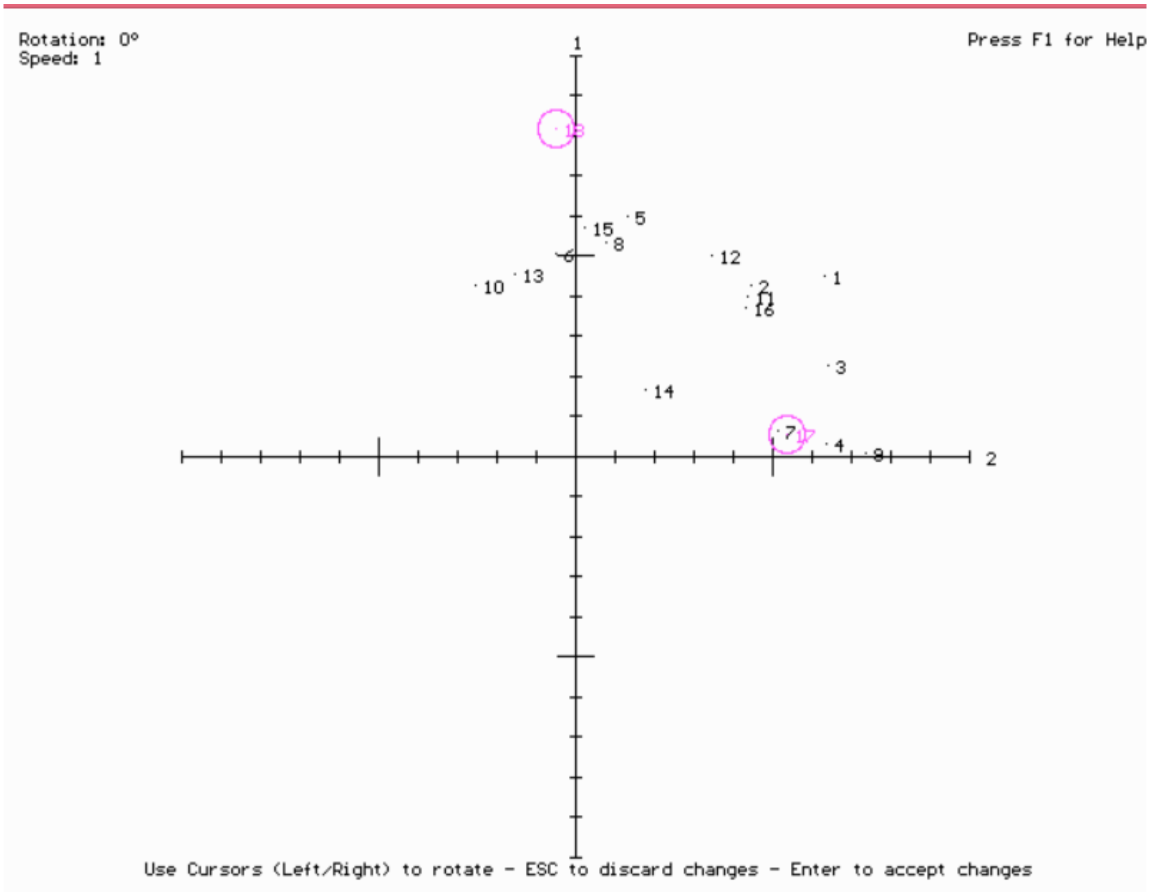
Schommer (1990) developed her Likert-scale survey based upon prior studies that involved extensive interviews. R-factor analysis determined that the structure of her 63-item questionnaire includes four factors, which represent the stability of knowledge, the structure of knowledge, the speed of learning, and the ability to learn. This structure has been repeatable across a variety of domains (Schommer & Walker, 1995) and ages (Duell & Schommer-Aikins, 2001; Schommer, Calvert, Gariglietti, & Bajaj,1997; Schommer-Aikins, Brookhart, & Mau, 2000; Schommer-Aikins, Duell, & Hutter, 2005). An earlier study used a Q-sample that consisted of 32 of the 63 statements from Schommer's (1990) Likert-scale survey selected across the four domains (factors) described by her. However, the results of this earlier study indicated that the wording led to students sorting based upon what they viewed as a socially acceptable response rather than their actual opinions. Student comments and post-sort interviews also indicated problems. The researchers decided to change wording to make statements more specific to individuals (e.g., use of "I" rather than "students") and the need for additional statements pertaining to students' epistemological beliefs. Twelve statements (from across the four domains) were added to the reworded, original 32 statements to create an improved Q sample with 44 statements. Further details about the development and selection of the Q sample are available in Ramlo (2006/7, 2008a, 2008b).

Students sorted a Q sample of 44 statements based upon their views of learning in their particular course during the final exam period. In addition, their instructors sorted these same statements with two different conditions of instruction: an ideal student view of learning in their course and a typical student view of learning in their course. Student and instructor sorts were analyzed together for each course. Centroid extraction followed by theoretical (hand) rotation was used because scientifically exploring the instructors' views relative to their student views represented the purpose of the study. For simplicity, the details of only one of the course's analyses (physics) are provided but the discussion does include interesting findings from one of the other courses.

For the first semester physics course, 16 students sorted at the final exam. PQMethod (Schmolck, 2002), software specially designed for entering and analyzing Q sorts (Newman & Ramlo, 2010), was used for the analyses. Because the investigation was focused on not only how students thought about their physics course learning experiences but also how they compared to the "Ideal" and "Typical" student views of the instructor, the researcher chose centroid with theoretical rotation. The rotation purposefully examined where the "Ideal" and "Typical" Q sorts were in relation to each other but also in relation to the students' sorts. Using centroid with theoretical rotation led to two distinct factors each with one of the instructor sorts. It is worth noting that when PCA with Varimax was used, with no limit to the number of factors to extract, eight populated factors emerged with the "Ideal" sort not represented by a factor. When the researcher only allowed two factors to be extracted, the factor structure was similar to the results using centroid with theoretical rotation. Specifically, with two factors using PCA and Varimax, each factor was represented by seven sorts and the descriptive results were similar to the ones we discuss here. However, the selected solution with centroid with theoretical rotation allowed the researcher to not only explore the factor structure but the descriptive results using abductive reasoning to investigate best theoretical solutions. During the abductive process, the researcher investigated multiple factor solutions while considering the descriptive results in conjunction with written post-sort responses and her knowledge of the students and instructor. The centroid solution with theoretical rotation that was selected as the best solution, based on theoretical significance, had two factors represented by eight sorts on Factor 1 (including the "Typical" student sort) and six sorts on Factor 2 (including the "Ideal" student sort). For the four remaining student sorts, three had mixed loadings on both factors and one was not represented by either factor.

Figure 1 contains a screen shot of the final theoretical rotation between the two factors. The points with the larger circles around them represent the "Ideal" (17) and "Typical" (18) instructor sorts with the "Ideal" closest to the 2-axis (X-axis, representing the relationship to Factor 2) and the "Typical" closest to the 1-axis (the Y-axis, representing the relationship to Factor 1). The correlation of the "Ideal" sort with Factor 1 is 0.82 and the correlation between the "Typical" sort with Factor 2 is 0.54. By ensuring that these instructor sorts were correlated with a factor, the researcher could then examine the student views relative to these instructor views.

Figure 1: Final Theoretical Rotation between the Two Factors



To investigate how the two perspectives (factors) differ within this factor solution, the discussion will focus on the distinguishing statements. Distinguishing statements are those statements that differentiate the two perspectives at a statistical significance level of .05. Those statements that distinguish Factor 1 (Typical) from Factor 2 (Ideal) are provided in Table 1.

Table 1: Those statements that distinguish Factor 1 (Typical student) from Factor 2 (Ideal student)

No.	Statement	Factor 1 Grid Position	Factor 2 Grid Position
8	Working with classmates inside &/or outside this class helps me learn.	5	4
41	Problems on tests and quizzes should be ones we've seen before in class.	4	1
9	Learning something really well takes me a long time.	4	-1

35	I am struggling to learn in this class.	3	-1
2	I need to learn how to study more effectively to succeed in this class.	2	-1
3	Working hard on difficult problems does not help me learn in this class.	2	-3
25	Doing homework helps me learn in this class.	2	5
15	It's a waste of time working on problems which have no clear-cut and unambiguous answer.	2	-2
19	Thinking about what a textbook says is more important than memorizing what a textbook says.	2	3
38	If I get the right answer for a problem I should get all of the points, no matter how I got the answer.	1	-1
14	Sometimes I just have to accept answers from my teacher even though I don't understand them.	1	0
17	I will get mixed up if I try to combine ideas from this class with what I already know.	1	-2
5	I need to learn how to learn.	1	-5
22	When I study, I try to get the big ideas instead of focusing in on the details.	0	3
40	I learn from the mistakes I make in this class.	0	3
27	I like the exactness of math-type subjects.	0	1
39	This class makes me think about things differently.	0	3
1	What I learn from textbook depends on how I use it to study.	-1	0
42	My instructor's expectations for my learning in this course are not reasonable.	-1	-3
33	If I regularly come to class that should be enough to pass this class.	-1	-4
21	I reorganize the information from this course so it makes sense to me.	-2	1
24	I feel comfortable applying what I learned in this class to the real-world.	-2	2
20	I try my best to combine information across chapters in this class or even from other classes.	-2	4
34	I am genuinely interested in learning the topics in this course.	-3	1

26	Reading the textbook helps me learn in this class.	-3	-1
29	What I learn in this class will help me in other classes.	-3	2
30	What I learn in this class will help me when I get a job in my field.	-4	1
36	I would learn more in this class if I spent more time reading the book.	-5	-1

By examining the distinguishing statements, the Factor 1 "Typical student" view can be described as representing those students who are not seeking a challenging course (e.g., #41, "Problems on tests and quizzes should be ones we've seen before in class" at +4). Although engineering and engineering technology have their roots in physics, the "Typical student" view does not see the course as relevant (#29, "What I learn in this class will help me in other classes" at -3; #30, "What I learn in this class will help me when I get a job in my field" at -4). They are neutral (#42 at -1) about whether the instructor's expectations for their learning are reasonable. These students are struggling to be successful in this physics course (e.g. #35 "I am struggling to learn in this class" at +3).

In contrast, the "Ideal student" view feels that the instructor's expectations are reasonable (#42 at -3) and are neutral about whether they are struggling in to be successful in the physics course. The "Ideal student" view does not think that coming to class alone should ensure a passing grade (#33 at -4), something the "Typical student" view feels neutral about (-1). The Ideal students believe (+5) that doing homework helps them learn whereas the "Typical" students are unsure (+2). The "Ideal" students try their best to combine information across chapters or even from other classes (#20 at +4), whereas the Typical student felt more neutral at -2, and try to learn from their mistakes (#40 at +3 with Typical at 0). The Ideal student focus on learning is exemplified by statement #5 (I need to learn how to learn) at position -5. This course helps these students think about things differently (#39 at +3). Together, the distinguishing statements indicate that the "Ideal" students already know how to be effective learners. The "Ideal" students appear more open minded than the "Typical" students and are focused on their learning in the course. In contrast, the "Typical" students are unsure whether or not they need to learn how to learn (+1). The "Typical" students are not as interested in seeing a big picture or making connections within the course; they present an attitude that is more about passing a course than what they will take away for future course work or their careers. Consensus statements, which represent agreement between the two factors/ perspectives, indicated that students felt they were at least somewhat responsible for their learning and had to exert effort to be successful in the course. Students agreed that doing homework was important for their learning. They also felt neutral about the usefulness of the textbook. Because the two factors included the instructor's "Typical" and "Ideal" student sort, the consensus represents agreement among the students' and the instructor's views.

Recall that the purpose of this study was to examine the relationship between the instructor's ideal and typical student views and the views reported by the students enrolled in the course. Thus it is first interesting that one set of seven students are aligned with the "Typical" view of the instructor and another five are aligned with the "Ideal" view of the instructor. Each of these views is unique, as demonstrated by the distinguishing statements in Table 1. The study revealed that the instructor's conceptualization of "Typical" and "Ideal" student correlated with two unique groups of students. Theoretical rotation was necessary to ensure that the instructor's two views about students were represented by factors but also that students also populated those factors. Factor extraction with centroid ensured that the explorations with hand rotation did not violate the assumption of a one, best mathematical solution while also preserving the interrelationships of the sorts. Recall that the rotation of the factors actually is a rotation of the perpendicular axes while the Q sorts remain fixed, preserving their relative positions to one another (Brown, 1980; Stephenson, 1953).

The analyses for the other two courses within the larger study also benefited from the use of centroid with theoretical rotation. In these cases, theoretical rotation also ensured that instructor views were revealed relative to their student views. Certainly not every case included a factor structure like that of the physics course and its instructor. For instance, in the Machine Design III course, the instructor's "Typical" and "Ideal" student were on the same factor but with one having a negative correlation and the other a positive correlation with that factor/perspective. In other words, this instructor perceived his typical student as one that was the polar opposite of his ideal. The two other factors/ perspectives that emerged within that course's students were populated by students only.

Thus, the larger study provided insight for instructors as they reflected on their "Typical" and "Ideal" sorts relative to their actual students' views. Follow up interviews with the instructors demonstrated that the study made them more keenly aware of their views of students including certain biases they were not aware of previously. In order to demonstrate that there are other studies with meaningful results because of a researcher's abductive use of centroid with theoretical rotation, two more examples from the literature are shared and briefly discussed.

Example 2: Psychiatric Ward

Brown (1978, 1980) recounts a multi-participant study where a factor with only one correlated sort provided important theoretical insight but would not meet an R methodological view of statistical importance. Specifically, statistical significance is often associated with a factor eigenvalue of 1.00 or greater; yet in this study important information would have been lost if the researcher had not investigated the theoretical significance of this factor. This particular study concerns decision making within a team working in a psychiatric ward. The fourth of four factors represents only one person. This factor's eigenvalue is 0.97, less than the 1.00 minimum used in R factor analysis studies. However, this fourth view is of great theoretical importance because the ward physician defines this factor. This physician was the ward's team leader and made the key decisions in the ward. Without this fourth factor, the decision making climate within the ward would be incomplete. Fortunately, the Q researcher here was interested in the practical, theoretical significance of the factors, not the statistical significance. Thus within the interpretations of the factors, the ward physician's view could be contrasted with those of the nurses, social worker, psychologists, and aides, with a major conflict revealed within the factor structure. The means of arriving at the factor structure, by the way, was apparently seen as inconsequential because the author does not indicate the means of factor extraction or rotation within this study. However, the theoretical significance of the findings is evident nonetheless. In addition, if the ward's physician had not been represented by a single factor, as can be facilitated through theoretical rotation, a major aspect of the decision making climate within the ward would have been lost. A similar situation exists within the next, single-case study.

Example 3: Single-case Study and Theoretical Significance

The single-case study by McKeown (1975) is summarized within Brown (1978). In this study, a disturbed young woman sorted the same Q sample multiple times based upon how she and others saw her. The fourth and fifth factors of this study each had one sort representing that factor/view. However, these factors represented the views of the subject's father and mother, respectively, with the mother's factor eigenvalue as 0.74. The factor structure reveals the subject's situation more clearly and without the fourth and fifth factors, her sense of failure regarding her parents' expectations would not have been revealed. Like the ward physician study above, the McKeown study does not describe the process of factor extraction and rotation used. One can presume, however, that if the patient's representations of her mother's and father's views had not emerged from the factor analysis process, theoretically significant findings would have been lost. Both of these examples exemplify the idea of Q seeking theoretical significance within the factor analytical stage, rather than seeking simple structure and other mathematical/ statistical goals. This is why the researcher's abductive reasoning and sense of inquiry are important within the exploration of factor solutions in Q.

Summary

All of these studies offer examples of what Stephenson (1984) described as "statements of problems" rather than "statements of facts" where researchers' use of theoretical rotation served them well. He posits that the goal of rotation is to provide factors that are operant and centroid extraction with theoretical rotation may be necessary to provide insight. Stephenson also offers several examples and reminds us that "one cannot take liberties with data" (p. 89) and that factor analysis in Q is about the advancement of knowledge within a quest for "concepts of importance" (p. 90).

Conclusions

It is important to understand that, as a student of Spearman, Stephenson was well versed with all of the statistical mandates associated with R factor analysis. Yet Stephenson rejected these statistical requirements, deeming them inconsequential in Q (Brown, 1980) for reasons detailed within this manuscript. As Thompson (1962, p. 215) stated, *the mathematically exact solution need not necessarily mirror reality*. Although some deem the centroid extraction and theoretical rotation as antiquated, Stephenson's retention of these factor analytic preferences was purposeful, innovative, and pragmatic. Specifically, Stephenson took his stand about an old question within psychology about whether to seek mathematical and statistical precision or utilize a more inquiry-based approach to knowledge generation (Thompson, 1962). He specifically designated his factor analytic preferences within Q methodology to scientifically explore subjectivity while not violating assumptions associated with other choices (e.g., PCA and Varimax and their singular best mathematical solution). Scientific inquiry is at the heart of scientific discovery and Stephenson's factor analytic preferences enable researchers to use abductive reasoning in order to seek factors which are operants and based upon the subjectivity provided by the Q sorts. Such scientific factor analytic explorations can focus on discovering meaning and developing theory while effectively interpreting and describing differences amongst the views that exist within the group or individual under study.

These methodological aspects of Q offer the ability to scientifically study subjectivity. Therefore, Q methodologists need to better communicate the reasons behind the factor analytic preferences within Q, providing sufficient information to counter typical arguments for choices that are based on seeking mathematical purposes perhaps more appropriate for R factor analysis, like simple structure, rather than a pragmatic factor analytic approach that focuses on developing knowledge and theory. An understanding of the historical developments within factor analysis and Q may facilitate those communications while also encouraging Q researchers to use factor analysis that use centroid extraction and theoretical rotation within Q studies. An understanding of the ontological, philosophical, and epistemological aspects of Q methodology are also necessary for understanding Stephenson's preferences at the factor analytical stage (Ramlo, 2015).

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