Connected Representations of Knowledge: Do Undergraduate Students Relate Algebraic Rational Expressions to Rational Numbers?

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The attainment and retention of later algebra skills in high school has been identified as a factor significantly impacting students' postsecondary success as STEM majors. Researchers maintain that learners develop meaning for algebraic procedures by forming connections to the basic number system properties. In the present study, the connections students form between algebraic procedures and basic number properties in the context of rational expressions was investigated. An assessment given to 107 undergraduate students in Precalculus that contained three pairs of closely matched algebraic and numeric rational expressions was analyzed. McNemar's test indicated that the undergraduate students' abilities related to algebraic rational expressions and rational numbers were significantly different, although serious deficiencies were noted in both cases. A weak intercorrelation was found in only one of the three pairs of problems, suggesting that the students have not formed connections between algebraic procedures and basic number properties.

The "Engage to Excel" report issued by the President's Council of Advisors on Science and Technology (2012) detailed the projected shortage of students graduating with science, technology, engineering, and mathematics (STEM) degrees in the United States. To meet projections, the United States will need to increase the current number of students who receive undergraduate STEM degrees by approximately 34% each year (President's Council of Advisors on Science and Technology, 2012). The report specifically identified the mathematics-preparation gap as a barrier to graduation for undergraduate students who major in a STEM field.

One reason very few students who start as STEM majors finish college with a STEM degree is their failure to succeed in the advanced mathematics courses that are required to earn a STEM degree (Rask, 2010; Snyder, Dillow, & Hoffman, 2008). Research studies have identified prior academic preparation in mathematics as a factor significantly impacting students' success in STEM majors (Astin & Astin, 1993; Kokkelenberg & Sinha, 2010; Post et al., 2010). Ost (2010) and Rask (2010) both identified grades students receive in introductory mathematics courses as a reason why students leave STEM majors. Likewise, Ehrenberg's (2010) review of five papers in a research symposium funded by the Sloan Foundation identified grades in introductory STEM courses along with prior academic preparation as the two most important factors that influence persistence in STEM fields.

Although the results should be considered carefully due to the age of the study, a 1993 study by Astin and Astin is perhaps the most comprehensive examination of factors influencing STEM persistence ever undertaken and is often cited in the undergraduate STEM education literature (e.g., Adelman, 1998; Elliott & Strenta, 1996; Harwell, 2000; Kokkelenberg, 2010; Lewis, Menzies, Najera, & Page, 2009; Seymour, 2002; Seymour & Hewitt, 1997; Vogt, 2008). Astin and Astin examined longitudinal data for 27,065 freshmen at 388 four-year colleges and universities and found that the strongest predictor of STEM persistence after four years of college was the students' entering level of mathematics or academic competency as measured by scores on college entrance exams, the American College Testing (ACT) Program Assessment or Scholastic Aptitude Test (SAT), and high school grade point average. A more recent, larger scale study by Kokkelenberg and Sinha (2010) analyzed longitudinal data for approximately 44,000 students at a New York State University and found evidence that prior academic preparation as measured by advanced placement (AP) credits and SAT scores was a significant indicator of success in a STEM major. Research has clearly identified grades in early STEM courses and prior academic preparation as two important factors influencing success in STEM majors. Mathematics educators may contribute to solving the STEM persistence problem by seeking to understand why students struggle in early postsecondary mathematics courses.

The mathematics entry point for STEM majors is often Precalculus (Post et al., 2010), and failure to succeed in this course is often a barrier for students continuing to study a STEM field (Adelman, 1998). Students' lack of algebraic manipulation skills is among several difficulties with Calculus that Tall (1993) observed. Similarly, in a study by Baranchik and Cherkas (2002), success in Precalculus was found to depend on the students' "later algebra skills," a term used to describe those algebra skills learned just before Precalculus. Therefore, it seems logical that one potential avenue for supporting the retention of STEM majors lies within the acquisition of later algebra skills.

A review of Precalculus textbooks supports the fundamental belief that the lack of algebra skills limits students' success in Precalculus. In fact, in five of the seven Precalculus textbooks selected for examination the author included an algebra review section (see Table 1). The algebra topics most often covered in these review sections included real numbers, exponents, factoring polynomials, rational expressions, and radicals. Proficiency with the procedures used to simplify or perform operations with rational expressions is also frequently identified by educators as a gap in students' academic preparation for college (Dawkins n.d.; Schechter 2009; Scofield, 2003). Mathematics instructor websites often identify simplifying and performing operations with rational expressions as a particular area of weakness for many students (Dawkins, n.d.; Schechter 2009; Scofield, 2003). Scofield (2003), a mathematics instructor for 14 years, published a comprehensive list on his course page of the most common algebra errors he witnessed students make in his Precalculus and Calculus classes. He observed that when faced with complicated algebraic fractions students tended to cancel everything in sight without regard to the fact that the numerator and denominator must first be factored. Schechter (2009) called this phenomenon "undistributed cancellations" and admitted that while he sees this error fairly often, he does not have a very clear idea of why it happens.

Table 1

Algebra Review Topics Found in Precalculus Textbooks

	Algebra Review Topics				
Textbook Author	Real Numbers	Exponents	Factoring Polynomials	Rational Expressions	Radicals
Young (2010)	X	X	X	X	X
Narasimhan (2009)	X	X	X	X	X
Zill & Dewer (2009)	—	—	—	—	—
Larson & Falvco (2010)	X	X	X	X	X
Cohen, Lee, & Sklar (2011)	X	X	X	X	—
Faires & DeFranza (2011)	—	—	—	—	—
Stewart, Redlin, & Watson (2011)	X	X	X	X	X

In an essay on Mathematics Education, Thurston (1990) noted that many calculus students made mistakes adding fractions, particularly symbolically. He noted that students commonly made this mistake: $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$. Similarly, researchers who reviewed a Precalculus Algebra course preceding Precalculus collected anecdotal evidence from faculty members and observed that students often had trouble simplifying or performing operations with rational expressions, especially when finding least common denominators and greatest common divisors (Karim, Leisher, & Liu, 2010). These findings identify the inadequate understanding of rational expressions as a limitation for many students.

The purpose of this study was to determine quantitatively if a correlation exists between students' abilities to simplify and perform operations with both rational numbers and rational expressions, with the goal of answering this question:

> To what extent, if any, is undergraduate students' ability to simplify and perform operations with algebraic rational expressions correlated with their ability to do the same with rational numbers?

The aim in this study was to establish the extent of undergraduate students' algebra deficiencies in the context of rational expressions and determine if a correlation exists between algebraic procedures and proficiency with rational numbers. The results of this study serve to inform future qualitative research which may provide much-needed insights into students' understandings and will inform the classroom practice of mathematics educators on a wide spectrum of educational levels; from the teaching and learning of fractions in elementary grades, to the teaching and learning of algebra at the secondary level, and to the teaching and learning of advanced mathematics courses at the postsecondary level. It is desirable for students to acquire algebra skills and establish an understanding of rational expressions at the earliest possible educational level, but many students will enter postsecondary education without this knowledge. It is critical that these algebra deficiencies in students be addressed so that the number of students succeeding in mathematics and persisting in STEM majors increases.

Background Literature

Empirical research specifically addressing algebraic rational expressions is limited. Most studies about students' misconceptions surrounding rational expressions typically investigate strategies students use to simplify the expressions, but do not address strategies used by students to perform operations such as addition, subtraction, multiplication and division (Constanta, 2012; Demby, 1997; Otten, Males, & Figueras, n.d.; Ruhl, Balatti, & Belward, 2011). Perhaps this void is due to the fact that the studies are typically situated in middle school or high school, and students at this level of education have not yet learned to perform operations with rational expressions.

In a study that examined the instruction of rational expressions in secondary school, Constanta (2012) found that teachers' predictions of the most common problems students would have simplifying rational expressions did not match the reality of the students' errors. During instruction, attention was not given to the composition of algebraic expressions and the relationship between operations in the numerator and denominator because teachers had assumed, incorrectly, that the students knew these concepts. Similarly, researchers who examined students' reflections of their procedures used to simplify rational expression problems were surprised at the widespread confusion about the meaning of "common factor," which was assumed to be common knowledge with undergraduate students (Ruhl et al., 2011).

Errors related to the cancellation of factors when simplifying algebraic rational expressions were also prevalent in studies by Constanta (2012), Otten et al. (n.d.), and Ruhl et al. (2011). Constanta (2012) hypothesized that cancellation errors resulted from the students' inability to perceive the numerator as a "whole" that is composed of different parts, while Otten and colleagues (n.d.) credited a misconception of the operation of division as the most likely cause of cancellation errors.

The Theoretical Framework of Connected Representations

According to Hiebert and Carpenter (1992), misconceptions and procedural errors can be understood in terms of connections. Hiebert and Carpenter's (1992) framework of connected representations provides a means for explaining students' understanding that is easily communicated and understood, and can shed light on both students' successes and failures. The idea from contemporary cognitive science that knowledge is represented internally, and that the internal representations are structured, is the primary assumption that supports the framework. Applying the cognitive science theory of internal representations to learning, Hiebert and Carpenter submitted that the construction of knowledge occurs when new information is connected to prior connections or when established connections are rearranged or abandoned. Knowledge from thickly connected networks provides a strong base for the construction of new knowledge, is quickly retrieved, and is more easily preserved over time. They proposed that this structure of connected representations is a useful way to describe mathematical understanding.

An important element in Hiebert and Carpenter's framework is the idea that mathematical procedures always depend on conceptual knowledge of mathematical principles. They proposed that connections formed between steps in a procedure are weak, but when they are linked to conceptual knowledge, the procedure becomes part of a larger network and then has access to all of the knowledge in that network, extending the range of the procedure's capabilities. Furthermore, they explicitly claimed that meaning for algebraic procedures is created when connections are formed between the procedure and the basic number properties. Hiebert further explained this idea in a later article (Hiebert &Wearne, 2003). Using rational expressions as a specific example, they stated that if students really understand what it means to add fractions, then adding rational expressions should just be an extension of that knowledge.

Using the theory of connected representations, then, one could argue that the algebraic processes of simplifying, adding, subtracting, multiplying, and dividing algebraic rational expressions are dependent on conceptual understanding of the basic number properties including, but not limited to, the commutative property for addition and multiplication, the associative property for addition and multiplication, and the distributive property of multiplication over addition.

Contemporary research supports the idea that proficiency with algebraic processes is dependent upon conceptual understanding of basic number properties specifically related to rational numbers. A study by Brown and Quinn (2007) found a positive relationship between proficiency with fractions and success in algebra. They concluded that understanding the structure of arithmetic could have a profound effect on learning the structure of algebra. Similarly, Wu (2001) contended, "the computational aspect of numbers is essential for the learning of both higher mathematics and science as well" (p. 13). Welder (2006) and Rotman (1991) also pointed to number knowledge of fractions as a prerequisite for learning algebra.

Methods

Research Context

This study took place at a Southeastern, public university that primarily serves in-state residents. Statistics from the fall semester of 2011 indicated that of the 26,442 students who were enrolled at this university, 73% were full-time, undergraduate students. The students with a declared STEM major made up 20% of the undergraduate population. While gender was almost perfectly balanced across the university, only 38% of undergraduate STEM majors were female. Minorities made up 29% of both the overall university population and undergraduate STEM majors. Of the 547 students who took Precalculus in the fall of 2011, approximately 48% made a grade of D or F, or withdrew from the class.

In the fall of 2012, the university offered fourteen sections of calculus and twenty-three sections of Precalculus, indicating that the first mathematics course for a majority of STEM majors at this university is Precalculus. Since the purpose of the study was to examine students' abilities with algebraic rational expressions and arithmetic rational expressions, the students taking Precalculus were the most appropriate population for the study.

Sample

The sample for this study was taken from the approximately 600 students enrolled in Precalculus during the Fall 2012 semester. Given the natural groups formed by the different sections of Precalculus, the most appropriate method of selection was cluster sampling (Kemper, Stringfield, & Teddlie, 2003). The university offered 23 sections of Precalculus with an average of 30 students in each section. Of the 23 sections of Precalculus, two evening sections were excluded from this study to maintain a homogeneous sample of full-time, traditional students. Thus, to obtain a sample size adequate for statistical analysis, five sections of Precalculus were randomly selected and students enrolled in those classes were invited to participate in this study. The students in this sample (n = 107) had an average age of 21. The gender distribution of the sample, 36% female, 59% male, 5% not reported, was aligned with that of the university. At 26%, the minority composition for this sample was slightly less than expected, but a large number of subjects, 30%, did not disclose their race or ethnicity.

Instrument

An assessment instrument developed by the researcher was used to collect data regarding students' procedural knowledge of algebraic and numeric rational expressions. The assessment instrument was limited to six mathematics questions to control for fatigue effects (Mitchell & Jolley, 2010) and to minimize each instructor's loss of instructional time. The assessment had two distinct sets of questions: one with three open-ended questions that asked students to perform operations with algebraic rational expressions, and one with three open-ended questions that asked students to perform operations with rational numbers. The assessment items were reviewed by two mathematicians to ensure that the content was valid for the purpose of measuring the students' ability to perform operations with rational expressions. The instrument also included demographic questions, which asked for the student's gender, age, major, ACT-mathematics score, highest high school math course taken, and race or ethnicity.

In creating this instrument, the researcher took care to design items that represented the important skills related to procedural knowledge of algebraic rational expressions. The researcher also considered the most common student errors reported in research when designing each assessment item (Demby, 1997; Otten et al., n.d; Ruhl et al., 2011). The rational number questions were designed to closely mirror the corresponding algebraic items, creating three pairs of matched items on the assessment.

The first set of problems (see Figure 1) presented one algebraic and one numeric rational expression with three terms and the operations of addition and subtraction. The denominators in this problem set, hereafter referred to as "Problem Set A," shared no common factors. The second set of problems (see Figure 2) presented one algebraic and one numeric rational expression with two terms and the operation of division. Before the numerator and denominator are correctly factored, this problem set, hereafter referred to as "Problem Set B," presented common terms in the numerator and denominator. The presence of common terms has been found to be a strong visual cue that leads students to inappropriately cancel terms as they would factors (Otten et al., n.d.). When the numerator and denominator in Set B are correctly factored, two common factors can be eliminated. The third set of problems (see Figure 3) presented one algebraic and one numeric rational expression with two terms and the operation of addition. The denominators in this problem set, hereafter referred to as "Problem Set C," shared one common factor.

$\frac{1+6}{2}+\frac{9}{2+3}-3$	$\frac{x+2}{4}+\frac{6x}{x+1}-3$

Figure 1. Numeric and Algebraic items in Problem Set A

$\frac{3}{3-1}\div\frac{9}{9-1}$	$\frac{x}{x-1}\div\frac{x^2}{x^2-1}$

Figure 2. Numeric and Algebraic items in Problem Set B

$\frac{1}{4\cdot 5}+\frac{3}{5\cdot 7}$	$\frac{1}{x^2-x-2}+\frac{x}{x^2-7x+10}$

Figure 3. Problem Set C

The order in which items are presented can affect responses (Mitchell & Jolley, 2010). Therefore, the numeric and algebraic items were presented in a random order on the assessment. The directions for the assessment instructed students to perform the given operations, show all of their work, and write their answer in simplest terms.

The assessment instrument was pilot tested with a convenience sample of high school students who had successfully completed Algebra II, the high school course in which rational expressions are taught. Although the students did not perform well on the pilot instrument, the students' work provided valuable information regarding their mathematical abilities. The pilot confirmed that the design of each item, the length of the assessment, and the assessment's level of difficulty were appropriate for use with post secondary students in a Precalculus course.

Procedures

In this study, the knowledge of rational expressions with which students enter college was investigated, and so it was important that the assessment instrument be given before related material was reached in their Precalculus courses, typically in chapter two of this university's approved text. During the first week of the semester, the researcher randomly selected five sections of Precalculus and contacted the corresponding instructors. Of these five instructors, three agreed to allow the researcher to visit their Precalculus classes and the remaining two instructors declined. As a result, the researcher randomly selected two additional sections of Precalculus and both of these instructors agreed to allow the researcher to visit their classes.

During the weeks three and four of the semester, the researcher visited each participating class to administer the assessment instrument. The students participated voluntarily in the study, and were fully aware that no penalty or extra credit would be assigned based on their completion of the assessment. At the beginning of the class session, the researcher provided instructions for how to complete the assessment instrument and then allowed the students 20 minutes to complete the task. The students were instructed that calculators should not be used to complete the assessment.

Data Analysis

Each item on the assessment was a variable in the study and was scored as correct or incorrect. Each numeric item was analyzed with its corresponding algebraic item. A correct item was assigned a value of one and an incorrect item was assigned a value of zero. The researcher observed that students in each of the five classes generally finished well before the 20-minute time-limit. Therefore, it was assumed that if a student who voluntarily participated left one or two items blank it was because they found the problem too difficult and the item was scored as incorrect. Assessments from three subjects had three or more blank items. In this case it was assumed that the student chose not to fully participate and the assessment was excluded from the analysis.

Due to the dichotomous nature of the variables in the study, the non-parametric McNemar's test for marginal homogeneity was used to determine if a difference did exist in the distribution of values across the numeric and algebraic items in each problem set. It follows that the phi coefficient is the appropriate measure for determining the intercorrelation between the responses of participants (Sheskin, 2004).

Results

The purpose of the study was to quantitatively examine the relationship between students' abilities with algebraic rational expressions and rational numbers. Specifically, the study aimed to answer the question:

> To what extent, if any, is undergraduate students' ability to simplify and perform operations with algebraic rational expressions correlated with their ability to do the same with rational numbers?

A contingency table was created for each pair of items showing the number of correct and incorrect numeric and algebraic items. Problem Set A required the student to add 3 terms with no common factors. The percent of the students who correctly answered the numeric item was 48.6% compared to 6.5% who correctly answered the algebraic version of this item (Table 2). Overall, only 3.7% of the students correctly answered both items. McNemar's test for marginal homogeneity indicated that the distributions of different values across the numeric and algebraic problems were significantly different (X^2 $(1, N = 107) = 37.96, p < .001$). The intercorrelation for this pair of problems (φ = 0.045) was less than the lower bound of 0.10 for which a small effect size could be recognized.

Table 2

Comparison of Student Responses to Problem Set A

			Algebraic		
			Incorrect	Correct	Total
Numeric	Incorrect	Count	52	3	55
		Row %	94.5%	5.5%	100%
	Correct	Count	48	4	52
		Row %	92.3%	7.7%	100%
		Total	100	7	107

Problem Set B required the student to divide terms with two common factors. The percent of students who correctly answered the numeric item was 37.3% compared to 6.5% who correctly answered the algebraic version of this item (Table 3). Overall, 5.6% of the students correctly answered both items. McNemar's test for marginal homogeneity indicated that the distributions of different values across the numeric and algebraic problems were significantly different (X^2 $(1, N = 107) = 29.26, p < .001$). The intercorrelation for this problem (φ = 0.264) indicated a small effect size.

Table 3

Comparison of Student Responses to Problem Set B

			Algebraic		
			Incorrect	Correct	Total
Numeric	Incorrect	Count	66	1	67
		Row %	98.5%	1.5%	100%
	Correct	Count	34	6	40
		Row %	85.0%	15.0%	100%
		Total	100	7	107

Problem Set C required the student to add two terms with only one common factor. The percent of students who correctly answered the numeric item was 41.1% compared to 5.6% who correctly answered the algebraic version of this item (Table 4). Overall, 2.8% of the students correctly answered both items. McNemar's test for marginal homogeneity indicated that the distributions of different values across the numeric and algebraic problems were significantly different (X^2 $(1, N = 107) = 31.11,\ p < .001$). The intercorrelation for this problem (φ = .044) was less than the lower bound of 0.10 for which a small effect size should be recognized.

Table 4

Comparison of Student Responses to Problem Set C

			Algebraic		
			Incorrect	Correct	Total
Numeric	Incorrect	Count	60	3	63
		Row %	95.2%	4.8%	100%
	Correct	Count	41	3	44
		Row %	93.2%	6.8%	100%
		Total	101	6	107

Discussion

The results of this study suggest that undergraduate students have serious deficiencies with algebraic procedures in the context of rational expressions. Less than 14.0% of students correctly answered one or more of the algebraic rational expressions. In each of the three problem pairs, a significant difference was found in the distribution of scores, meaning the subjects have different abilities with algebraic and numeric problems. Although 69.2% of the students correctly answered one or more numeric items, the percentage of correct answers for each individual numeric item never exceeded 50%. Research has shown that proficiency with rational numbers is related to success in algebra (Brown & Quinn, 2007; Welder, 2006). Therefore, this result indicates that it is likely that deficiencies with rational numbers may also contribute to students' difficulties with college-level mathematics.

A correlation between students' abilities with algebraic and numeric rational expressions was found only in the division problem set, and then it could only be categorized as a small effect. The absence of medium or strong correlations between the algebraic and numeric items would suggest that although students were more likely to get a numeric item correct and the corresponding algebraic item incorrect, there is no relationship between their abilities in both contexts. The significance of these results should be considered within the limitations of this study. The reliability of the scores and validity of the instrument created for the purpose of this study could have been strengthened by the inclusion of more replications of the problem sets.

Research of algebraic procedures frequently mentions the connection between arithmetic and algebra (Herscovics & Linchevski, 1994; Linchevski & Livneh, 1999) first discussed by Thorndike in 1923. Hiebert and Carpenter's (1992) framework tells us that algebraic procedural knowledge is connected to conceptual knowledge of number properties. In previous studies, a disconnect between algebra and arithmetic was observed by researchers who concluded that the student errors they observed demonstrated a lack of operation sense (Otten et al., n.d.; Warren, 2003).

According to Skemp (1976), students are able to develop an instrumental understanding when we want them to develop relational understanding. If students have not developed a conceptual understanding of rational numbers, merely memorizing algorithms for adding, subtracting, multiplying, and dividing, they will not be able to make the connections between basic number properties and algebraic procedures described by Hiebert and Carpenter (1992). The poor performance of students on the rational number items may indicate they have only a superficial understanding of fractions. This observation may explain the results of this study, namely an unrelated difference in the students' abilities to perform operations with algebraic and numeric rational expressions. Furthermore, the small correlation seen between the numeric and algebraic division operations in this study may have occurred due to the students' consistency in application of the invert and multiply strategy in both the numeric and algebraic contexts. A larger variation in the choice of procedure between the numeric and algebraic contexts was observed in the problem sets with addition items (for more details, see Yantz, 2013).

Conclusion

Retention and graduation of STEM majors who will drive technology and science innovation is important to strengthening our nation's position in the global economy. Weak prior academic preparation in algebra often leads to low grades in introductory mathematics courses and discourages students from studying STEM fields. It is possible that helping students succeed in entry-level classes such as Precalculus could improve the retention and graduation of STEM majors. To this end, it is important to understand the conceptual and procedural knowledge that students have when entering college. This study established the existence and the extent of students' algebraic deficiencies with rational expressions, however additional research is needed to gain insights into what factors may influence students' difficulties with rational numbers and algebraic expressions. The future plans for this research include the qualitative analysis of students' written work and task-based student interviews.

While it is important for all students to have algebraic procedural knowledge, it is critical for those who desire to be scientists, physicists, or mathematicians and will study advanced mathematics. It is possible that if the algebra deficiencies in students are identified and addressed early, the number of students succeeding in mathematics and persisting in STEM majors will increase. Understanding students' conceptions and misconceptions related to rational expressions and how they connect algebraic procedures to basic number properties is an important step towards being able to promote success for all students in introductory mathematics courses, but particularly for STEM majors who might otherwise leave a STEM field of study.

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